Summary and Conclusions

- ***Summary of Critical Concepts***
- ***The Future Role of Game Theory in the Design and Regulation of Dynamic Spectrum Access Networks***
- ***Topics for Further Study and Research***

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What does game theory bring to the design of cognitive radio networks? (1/2)

- A natural “language” for modeling cognitive radio networks
- Permits analysis of ontological radios
  - Only know goals and that radios will adapt towards its goal
- Simplifies analysis of random procedural radios
- Permits simultaneous analysis of multiple decision rules – only need goal
- Provides condition to be assured of possibility of convergence for all autonomously myopic cognitive radios (weak FIP)

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What does game theory bring to the design of cognitive radio networks? (2/2)

- Provides condition to be assured of convergence for all autonomously myopic cognitive radios (FIP, not synchronous timing)
- Rapid analysis
  - Verify goals and actions satisfy a single model, and steady-states, convergence, and stability
- An intuition as to what conditions will be needed to field successful cognitive radio decision rules.
- A natural understanding of distributed interactive behavior which simplifies the design of low complexity distributed algorithms

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Game Models of Cognitive Radio Networks

- Almost as many models as there are algorithms
- Normal form game excellent for capturing single iteration of a complex system
- Most other models add features to this model
  - Time, decision rules, noisy observations, Natural states
- Some can be recast as a normal form game
  - Extensive form game

- Normal Form Game
  - $\langle N, A, \{u_i\}\rangle$
- Supermodular Game $\frac{\partial^2 u_i(a)}{\partial a_i \partial a_j} \geq 0$
  –
- Potential Game $\frac{\partial^2 u_i(a)}{\partial a_i \partial a_j} = \frac{\partial^2 u_j(a)}{\partial a_j \partial a_i}$
  –
- Repeated Game
  - $\langle N, A, \{u_i\}, \{d_i\}\rangle$
- Asynchronous Game
  - $\langle N, A, \{u_i\}, \{d_i\}, T\rangle$
- Extensive Form Game
  - $\langle N, A, \{u_i\}, \{d_i\}, T\rangle$
- TU Game
  - $\langle N, v\rangle$
- Bargaining Game
  - $\langle F, v\rangle$

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Steady-states

- Different game models have different steady-state concepts
- Games can have many, one, or no steady-states
- Nash equilibrium (and its variants) is most commonly applied concept
  - Excellent for distributed noncollaborative algorithms
- Games with punishment and Coalitional games tend to have a very large number of equilibria
- Game theory permits analysis of steady-states without knowing specific decision rules

- Nash Equilibrium
- Strong Nash Equilibrium
- Core
- Shapley Value
- Nash Bargaining Solution

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Optimality

- Numerous different notions of optimality
- Many are contradictory
- Use whatever metric makes sense

- Pareto Efficiency
- Objective Maximization
- Gini Index
- Shapley Value
- Nash Bargaining Solution

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Convergence

- Showing existence of steady-states is insufficient; need to know if radios can reach those states
- FIP (potential games) gives the broadest convergence conditions
- Random timing actually helps convergence

Model	Sensitivity	Rate
Dynamical Systems	Apply Lyapunov's direct method (when possible)	No general technique
Contraction Mappings	Everywhere convergent	$\left\|a(t^k), a^*\right\| \le \frac{\alpha^t}{1-\alpha}\left\|a(t^1), a(t^0)\right\|$
Standard Interference Function Power Control	Everywhere convergent	$\left\|\mathbf{p}(t^k), \mathbf{p}^*\right\| \le \alpha^k \left\|\mathbf{p}(0), \mathbf{p}^*\right\|$
Finite Ergodic Markov Chain	Converges to distribution from all starting distributions	Transition matrix dependent
Absorbing Markov Chain	$\mathbf{B} = \mathbf{NR}$	$\mathbf{t} = \mathbf{N1}$
Normal Form Game	Convergence not defined	Convergence not defined
Mixed Strategy Strategic Form Games	Convergence not defined	Convergence not defined
Repeated Game	Assumes no adaptations	Assumes no adaptations
Myopic Repeated Game	Apply IESDS, FIP, weak FIP	Length of longest improvement path
Potential Game	All autonomously rational decision rules converge	Length of longest improvement path
Supermodular Game	All locally optimal decision rules converge	Length of longest improvement path

Noise

- Unbounded noise causes all networks to theoretically behave as ergodic Markov chains
- Important to show Lyapunov stabiltiy
- Noisy observations cause noisy implementation to an outside observer
  - Trembling hand

Model	Lyapunov Stability	Attractivity
Dynamical Systems	Apply Lyapunov's direct method (when possible)	Apply Lyapunov's direct method (when possible)
Contraction Mappings	Global	Global
Standard Interference Function Power Control	Global	Global
Finite Ergodic Markov Chain	No	No
Absorbing Markov Chain	No	Only if unique steady-state
Normal Form Game	Stability not defined	Stability not defined
Mixed Strategy Strategic Form Games	Stability not defined	Stability not defined
Repeated Game (assuming correct differentiation of punishment and deviation)	Yes	Yes
Myopic Repeated Game	Not implicit to model	Not implicit to model
Potential Game	Isolated potential maximizers are Lyapunov stable for all rational decision rules.	Attractive to potential maximizers if finite action space or finite step size.
Supermodular Game	Best response decision rules if unique NE	Best response decision rules if unique NE

Game Theory and Design

- Numerous techniques for improving the behavior of cognitive radio networks
- Techniques can be combined
- Potential games yield lowest complexity implementations
  - Judicious design of goals, actions,
- Practical limitations limit effectiveness of punishment
  - Observing actions
  - Likely best when a referee exists
- Policy can limit the worst effects, doesn't really address optimality or convergence issues

- Supermodular games
  - Steady state exists
  - Best response convergence
- Potential games
  - Identifiable steady-states
  - All self-interested decision rules converge
  - Lyapunov function exists for isolated equilibria
- Punishment
  - Can enforce any action tuple
  - Can be brittle when distributed
- Policy
  - Limits worst case performance
- Cost function
  - Reshapes preferences
  - Could damage underlying structure if not a self-interested cost
- Centralized
  - Can theoretically realize any result
  - Consumes overhead
  - Slower reactions

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Future Directions in Game Theory and Design

- Integrate policy and potential games
- Integration of coalitional and distributed forms
- Increasing dimensionality of action sets
  - Cross-layer
- Integration of dynamic and hierarchical policies and games

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Future Direction in Regulation

- Can incorporate optimization into policy by specifying goals
- In theory, correctly implementing goals, correctly implementing actions, and exhaustive self-interested adaptation is enough to predict behavior (at least for potential games)
  - Simpler policy certification
- Provable network behavior

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Avenues for Future Research on Game Theory and CRNs

- Integration of bargaining, centralized, and distributed algorithms into a common framework
- Cross-layer algorithms
- Better incorporating performance of classification techniques into behavior
- Asymmetric potential games
- Bargaining algorithms for cognitive radio
- Improving the brittleness of punishment in distributed implementations with imperfect observations
- Imperfection in observations in general
- Time varying game models while inferring convergence, stability…
- Combination of policy, potential games, coalition formation, and token economies
- Can be modeled as a game with to types of players
  - Distributed cognitive radios
  - Dynamic policy provider

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Questions?

Cognitive Radio Technologies

www.crtwireless.com

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