Game Theory in the Analysis and Design of Cognitive Radio Networks

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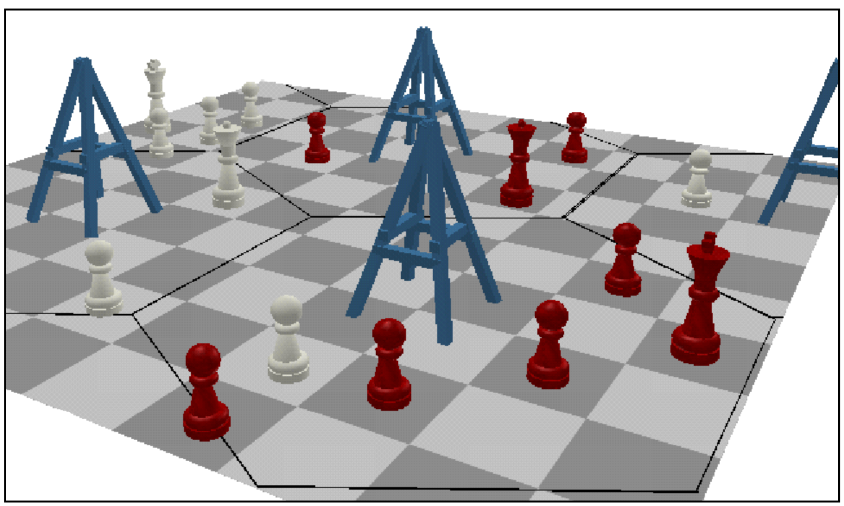
2014 Symposium and Summer School on Wireless Communications



Cognitive Radio Technologies
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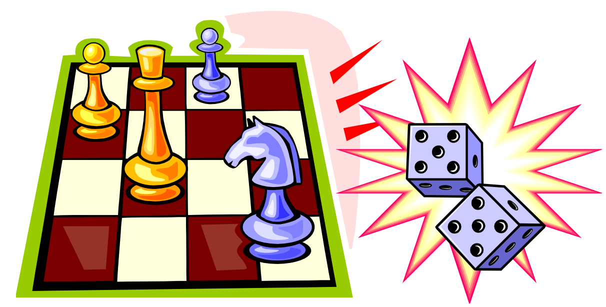
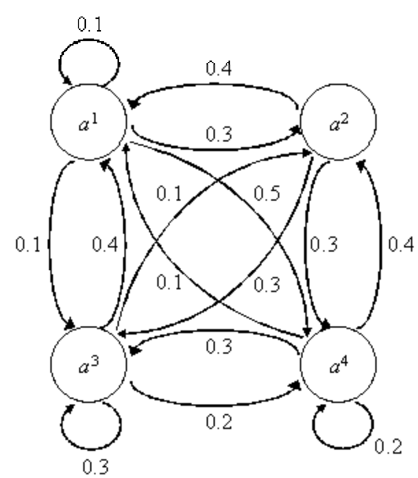
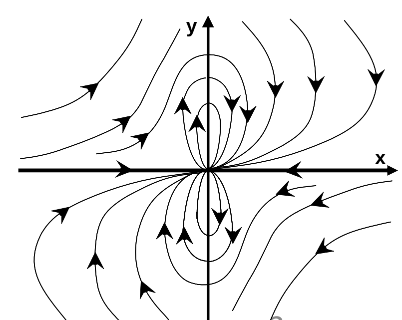
This is an **intermediate** level course. I'm assuming you have prior exposure to the following

- Cognitive Radio (if not, see Fette)
  - Basic implementation approaches
  - Typical applications
- Basic Game Theory (if not, see Dutta)
  - Typical content for senior level course
  - Nash equilibria, mixed strategies, zero sum games



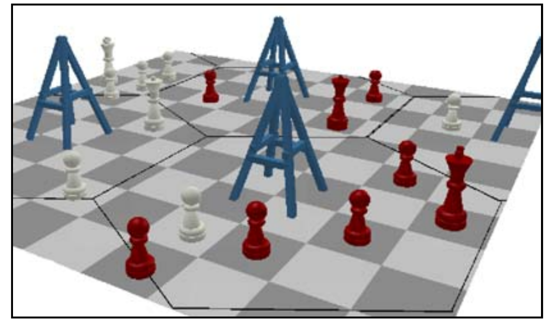
- Control Systems and Machine Learning
  - Lyapunov stability
  - Markov chains
  - Learning algorithms

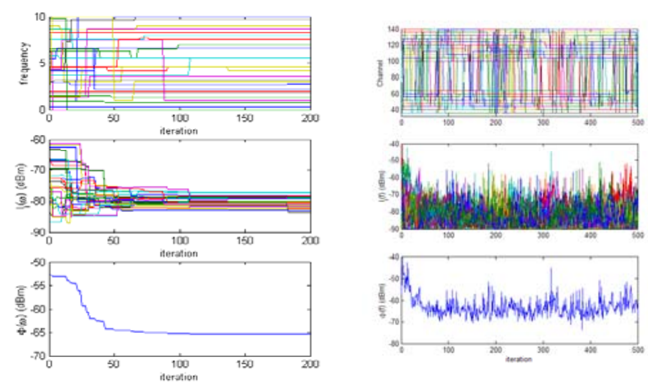


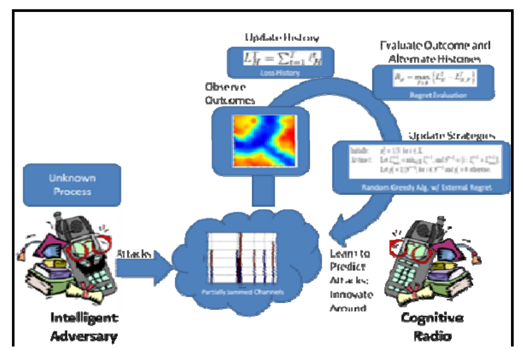


Course Material

- Why Apply Game Theory to Cognitive Radio Network Design?
- Potential Games for Distributed Radio Resource Algorithm Design
- Examples and Extensions
- Addressing Imperfections and Real World Effects
- Insights into adversarial games



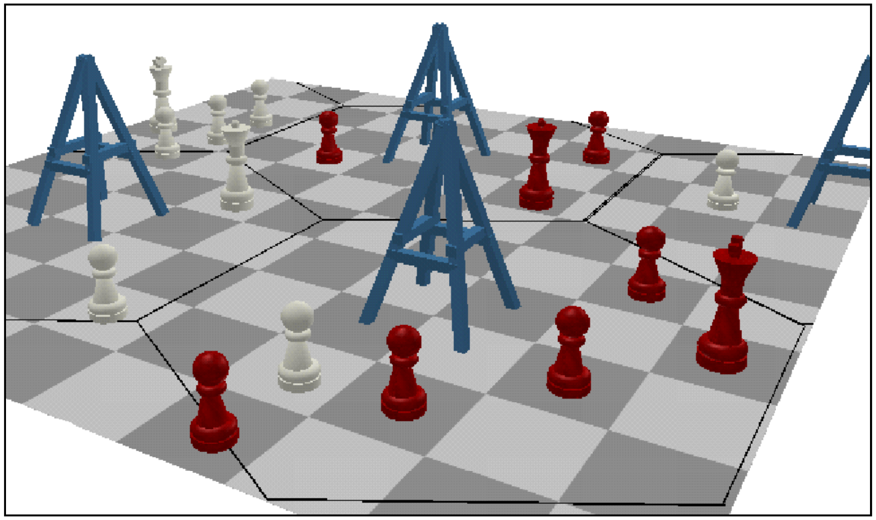




General Principles to Remember

- Realistic cognitive radio networks will have interactive decisions
- Inducing symmetry is good for stability
  - Finding win-win conditions is a close second
- Cannot generally (or even most of the time) start with arbitrary design objective functions
  - Instead use well-behaved solutions that correlate well
- Combining across layers is messy, but doable
  - Combinations with routing should exploit generalized congestion game
- Environmental effects can be accommodated with traditional good engineering practices
- Timing effects dominate game effects in adversarial scenarios
  - Your algorithms can be (and probably will eventually be subverted)

Why Apply Game Theory to Cognitive Radio Network Design?



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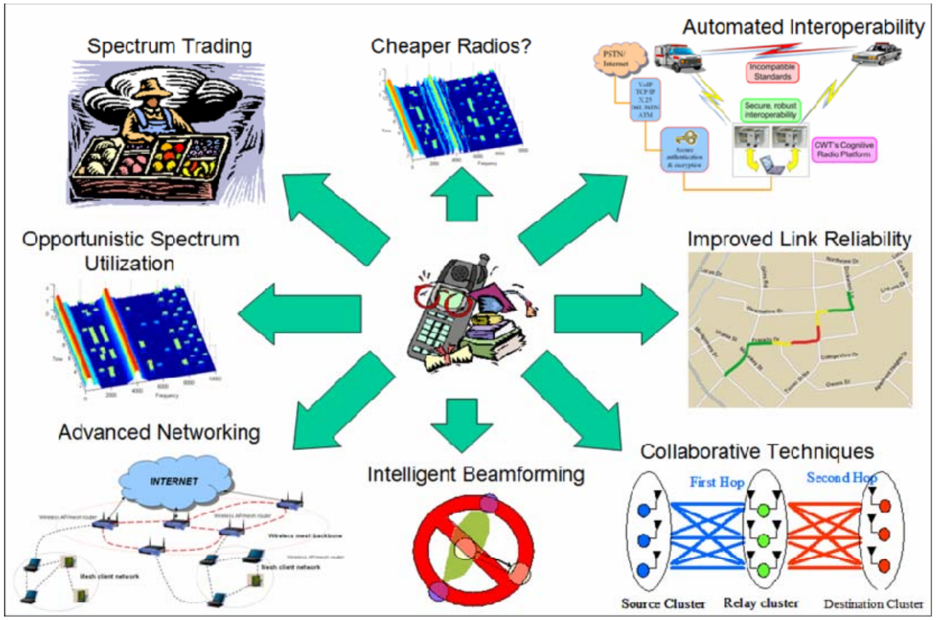
Cognitive Radio: Basic Idea

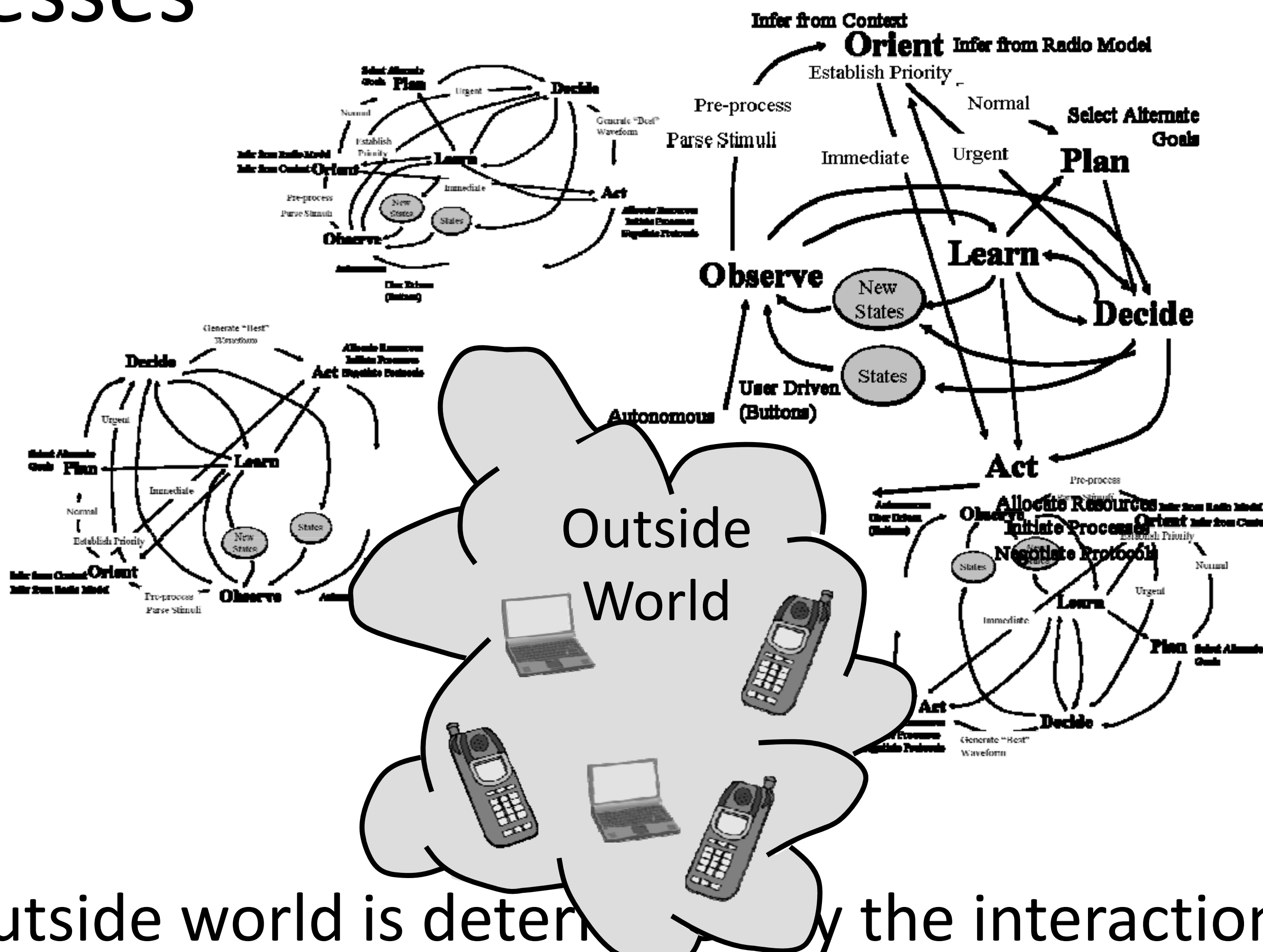
- Software radios permit network or user to control the operation of a software radio
- Cognitive radios enhance the control process by adding
  - Intelligent, autonomous control of the radio (link, network…)
  - An ability to sense the environment
    - Other information sources too
  - Goal driven operation
  - Processes for learning about environmental parameters
  - Awareness of its environment
    - Signals
    - Channels
  - Awareness of capabilities of the radio
  - An ability to negotiate waveforms with other radios



CR's Killer Apps

- Interference Management
- Context aware operation
- Taking humans out-of-the-loop
- Radio resource management
- Spectrum access

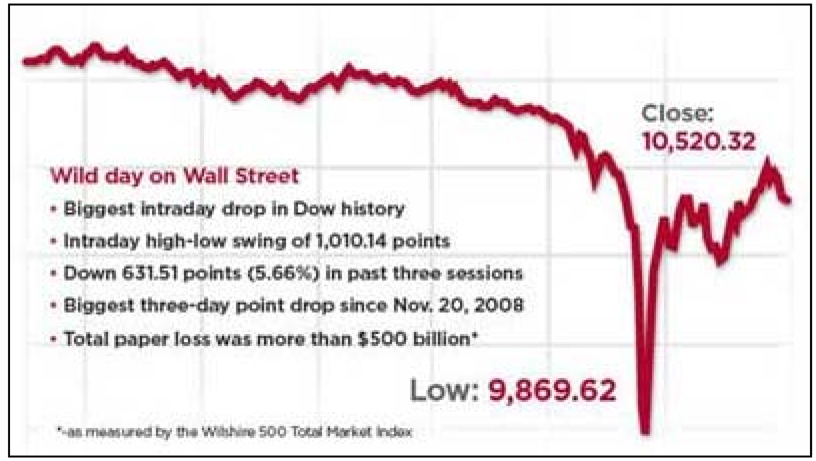
The Cognition Cycle and Interactive Processes



- Outside world is deter y the interaction of numerous cognitive radios

Issues Can Occur When Multiple Intelligences Interact

- Flash crash of May 6, 2010
  - Not just a fat finger
  - Combination of bad economic news, big bet by Universa, and interactions of traders and computers



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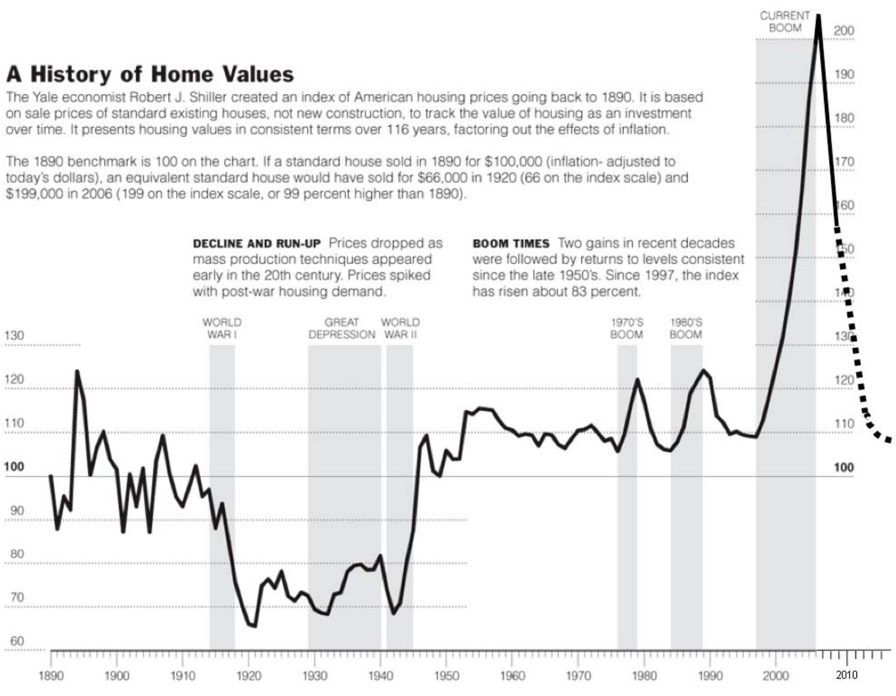

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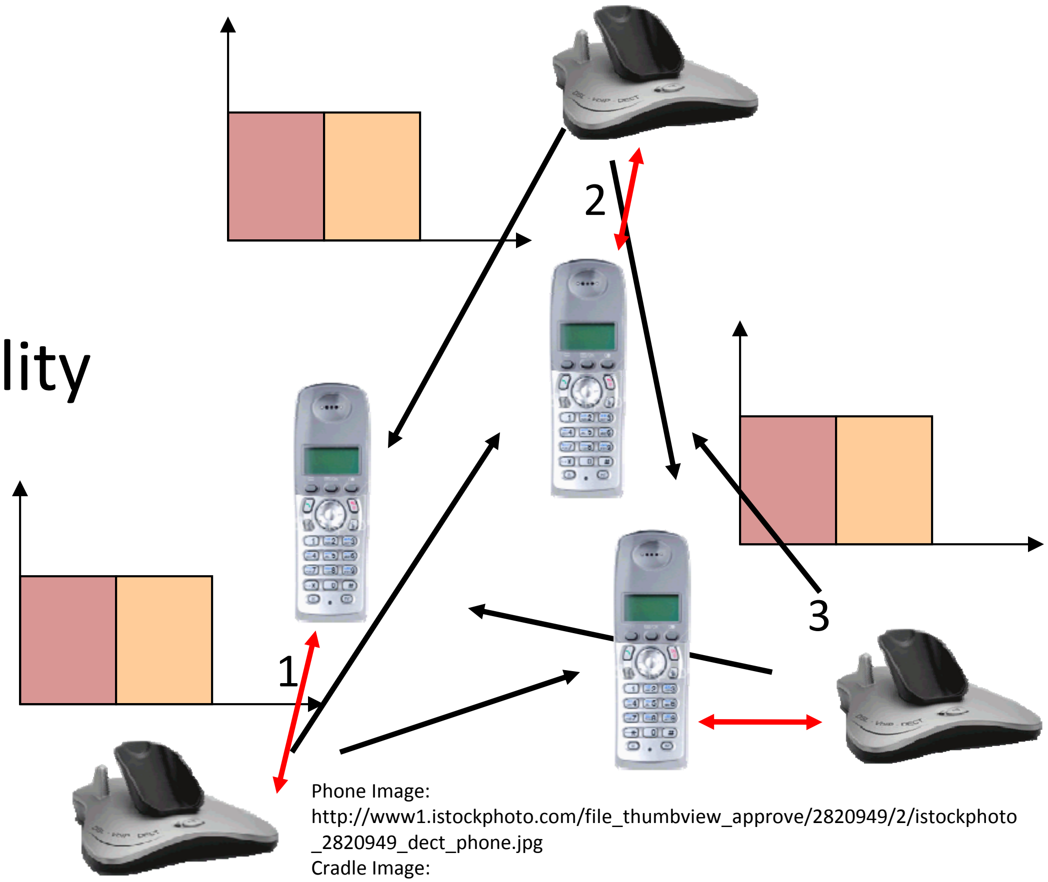
- Housing Bubble
  - Bounce up instead of down
  - Slower interactions lead to slower changes
  - Also indicative of the role beliefs play in instability

In heavily loaded networks, a single adaptation can spawn an infinite adaptation process

- Suppose
  - $g_{31}>g_{21}; g_{12}>g_{32}; g_{23}>g_{13}$
- Without loss of generality
  - $g_{31}, g_{12}, g_{23} = 1$
  - $g_{21}, g_{32}, g_{13} = 0.5$
- Infinite Loop!
  - 4,5,1,3,2,6,4,…



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Cradle Image:
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Interference Characterization

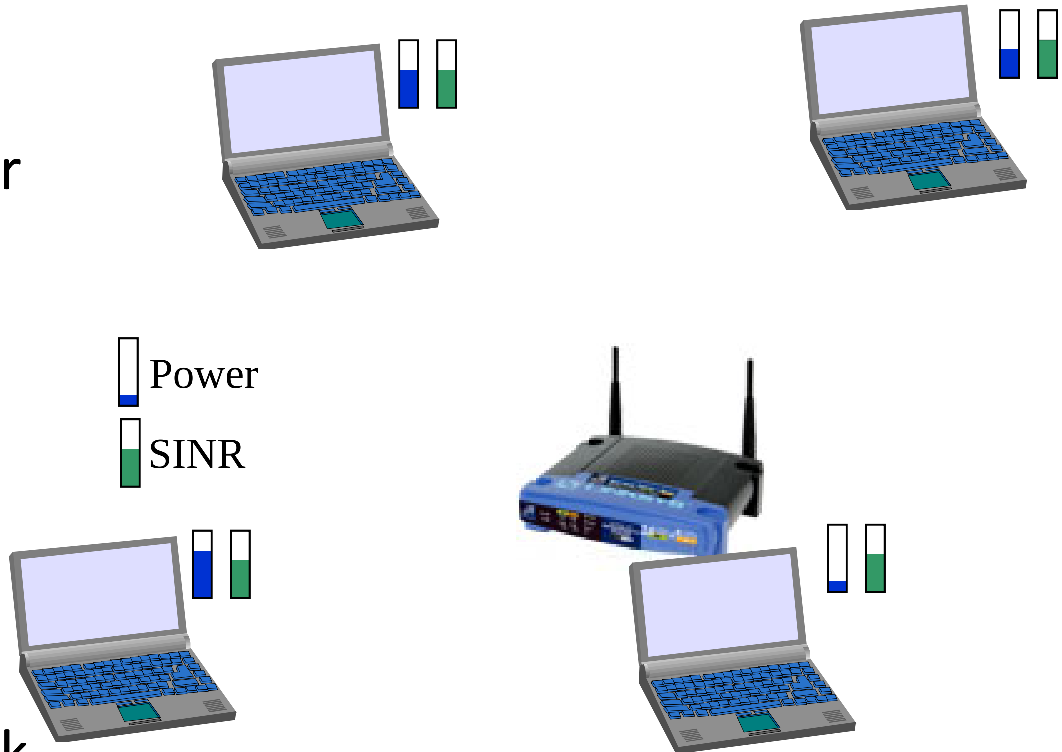
Chan.	(0,0,0)	(0,0,1)	(0,1,0)	(0,1,1)	(1,0,0)	(1,0,1)	(1,1,0)	(1,1,1)
Interf.	(1.5,1.5,1.5)	(0.5,1,0)	(1,0,0.5)	(0,0.5,1)	(0,0.5,1)	(1,0,0.5)	(0.5,1,0)	(1.5,1.5,1.5)
	0	1	2	3	4	5	6	7

Generalized Insights from the DECT Example

- If # links / clusters > # channels, decentralized channel choices will have a non-zero looping probability
- As # links / clusters $\rightarrow\infty$, looping probability goes to 1
  - 2 channels $p(loop) \geq 1-(3/4)^{{}^{n}C_{3}}$
  - k channels $p(loop) \geq 1-\left(1-2^{-k+1}\right)^{{}^{n}C_{k+1}}$
- Can be mitigated by increasing # of channels (DECT has 120) or reducing frequency of adaptations (DECT is every 30 minutes)
  - Both waste spectrum
  - And we're talking 100's of ms for vacation times
- "Centralized" solutions become distributed as networks scale
  - "Rippling" in Cisco WiFi Enterprise Networks
    - www.hubbert.org/labels/Ripple.html
- Also shows up in more recent proposals
  - Recent White Spaces paper from Microsoft

Locally optimal decisions that lead to globally undesirable networks

- Scenario: Distributed SINR maximizing power control in a single cluster
- For each link, it is desirable to increase transmit power in response to increased interference
- Steady state of network is all nodes transmitting at maximum power



[Insufficient to consider only a single link, must consider interaction](#)

Potential Problems with Networked Cognitive Radios

Distributed

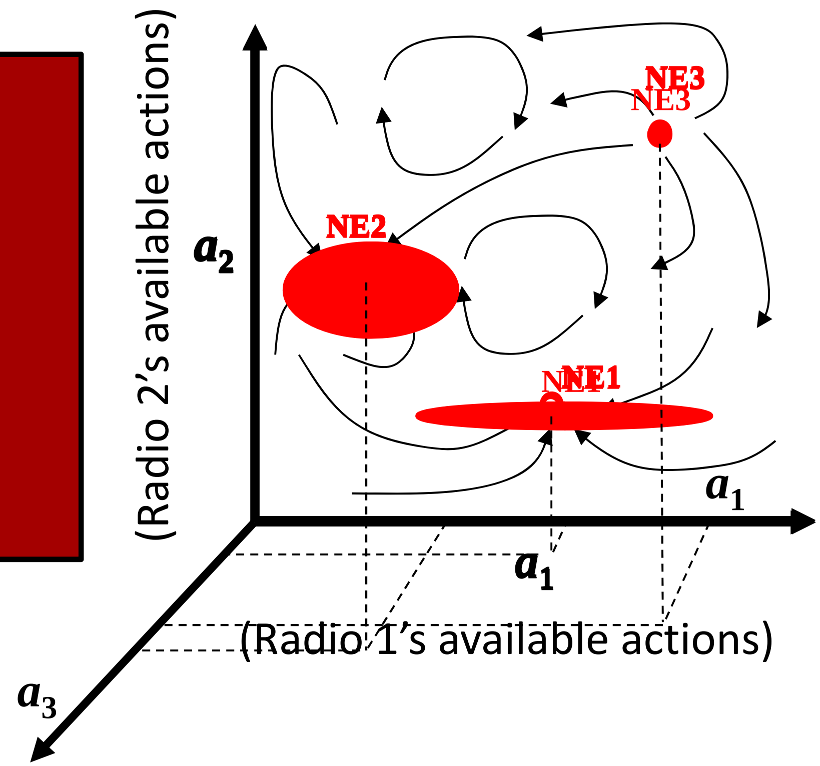
- Infinite recursions
- Instability (chaos)
- Vicious cycles
- Adaptation collisions
- Equitable distribution of resources
- Byzantine failure
- Information distribution

Centralized

- Signaling Overhead
- Complexity
- Responsiveness
- Single point of failure

Network Analysis Objectives

1. **Steady state characterization**
2. **Steady state optimality**
3. **Convergence**
4. **Stability/Noise**
5. **Scalability**



[illegible]Characterization

[illegible] state?

[illegible]eters?

[illegible]optimal?

Game Theory

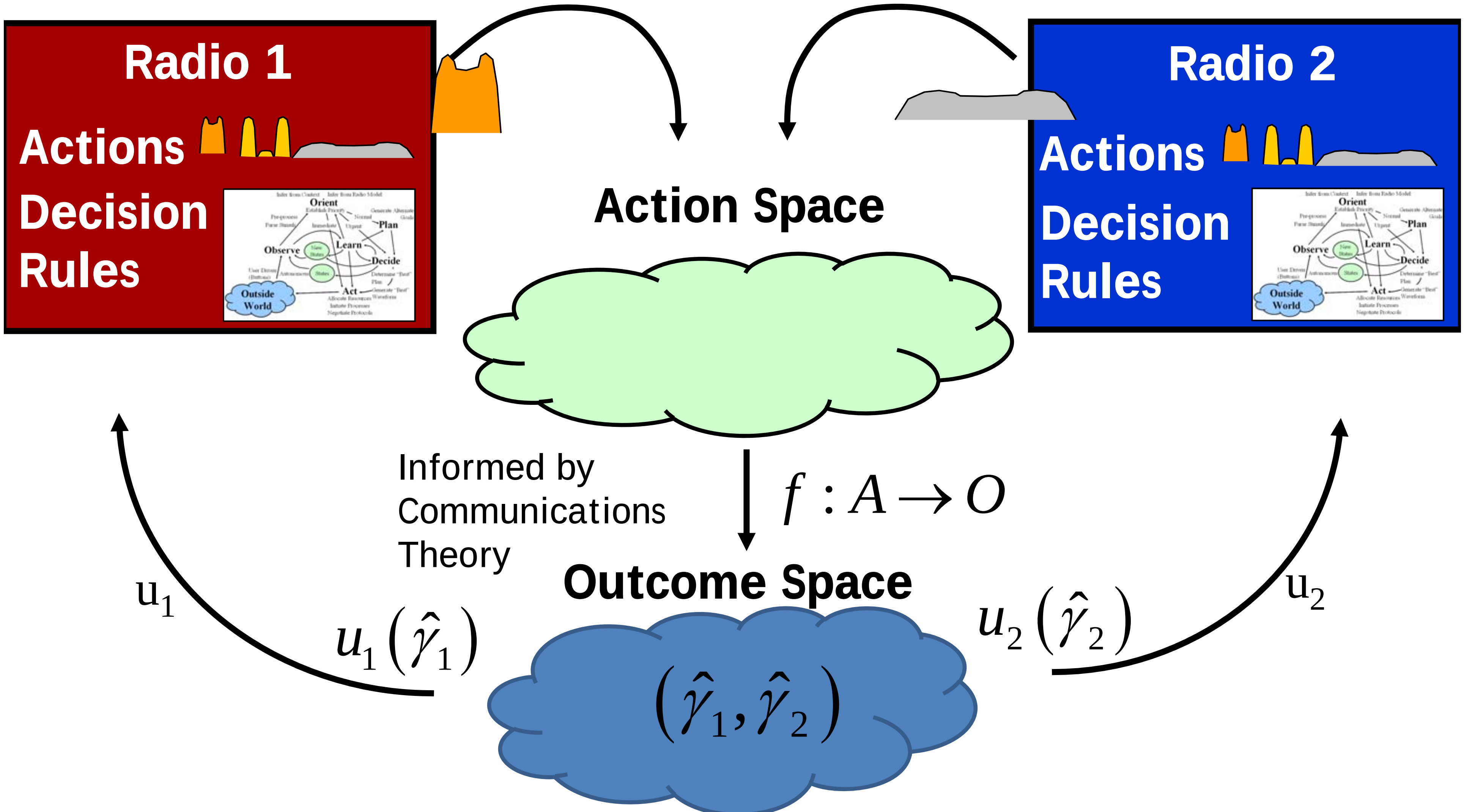
- The study of **mathematical models** of **conflict** and **cooperation** between **intelligent** **rational** **decision-makers** – Myerson

Basic Game Components

1. A (well-defined) set of 2 or more players
2. A set of actions for each player.
3. A set of preference relationships for each player for each possible action tuple.

- Games with just these three components (or a variation on the preference relationships) are said to be in **Normal** form or **Strategic** Form
- Preferences often represented as utility function for analysis but may not hold for people
- Some also introduce an outcome function which maps action tuples to outcomes which are then valued by the preference relations.

Interaction is naturally modeled as a game



Cognitive Radio Games

- Normal form game
  - Player + actions + utilities
  - One-off problems

	A	B
a	(3,3)	(1,5)
b	(5,1)	(2,2)

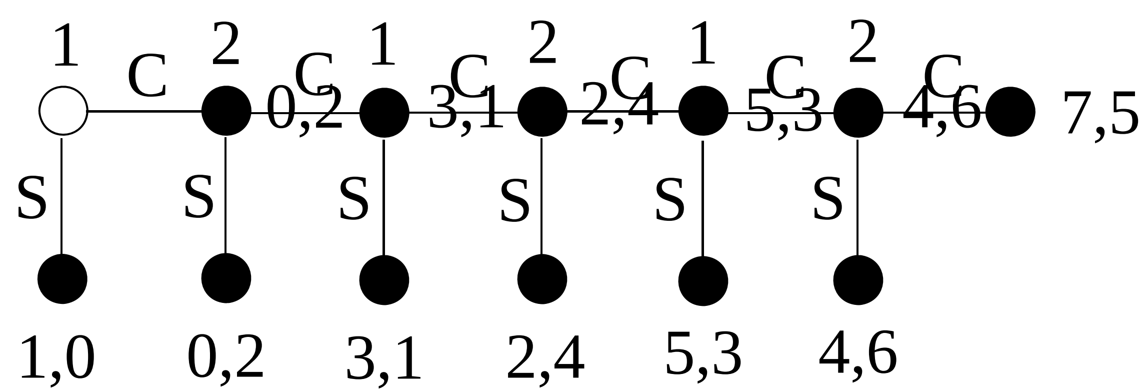
- Outcome Space Mapping
  - Relate utilities to actions
  - Need not be deterministic
- Timing
  - Extensive Form game
  - Repeated game

Game	⇔	Cognitive radio network
Player	⇔	Cognitive radio
Actions	⇔	Actions
Utility function	⇔	Goal
Outcome space	⇔	Outside world
Utility function arguments	⇔	Observations/orientation
Order of play	⇔	Adaptation timings

SINR as choice of channels

$$u_i\left(a_i, a_j\right) = \begin{cases} \dfrac{p_i g_{ii}}{N_i} & a_i \neq a_j \\ \dfrac{p_i g_{ii}}{p_j g_{ji} + N_i} & a_i = a_j \end{cases}$$

Alternating Packet Forwarding Game



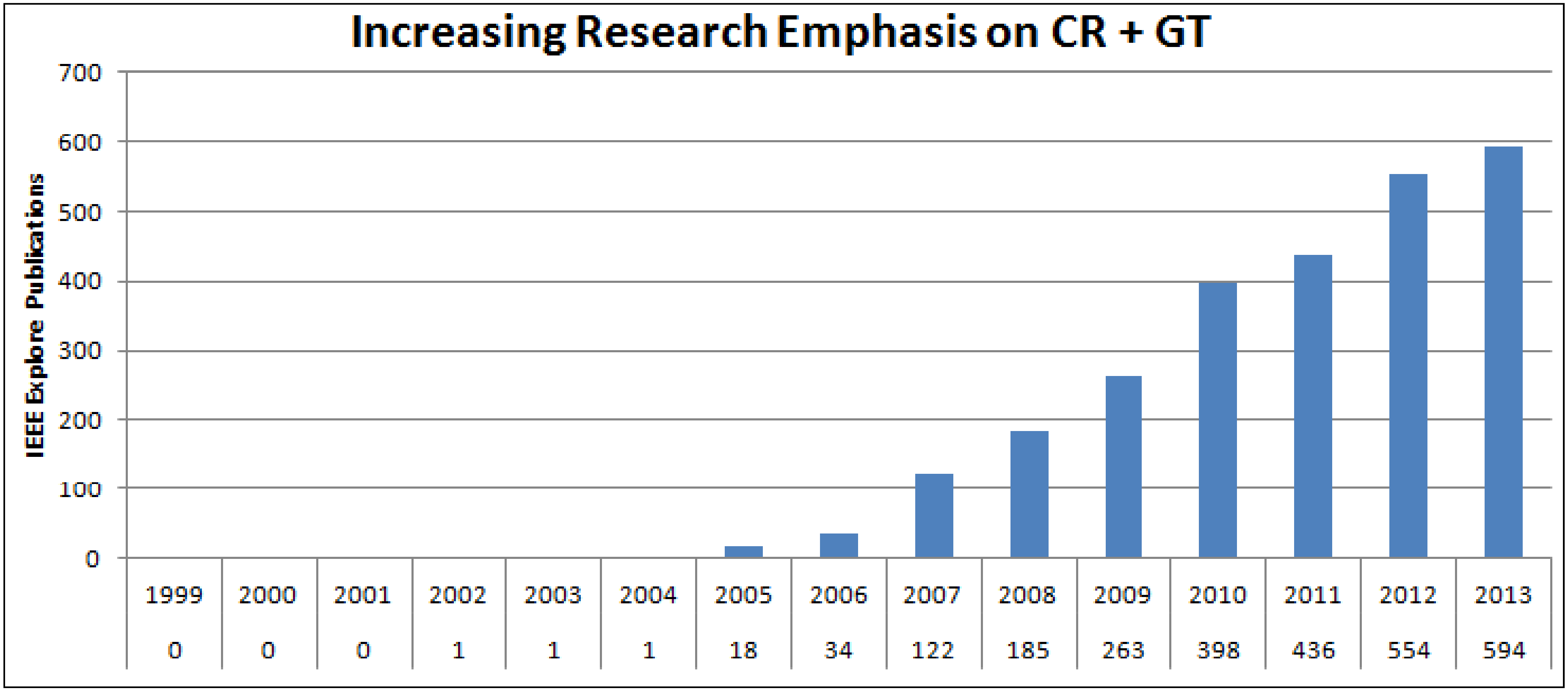
Common game-theory based approaches to designing CR nets

Approach	Benefits	Drawbacks
Repeated games with punishment	• Can enforce convergence to virtually any operating point • Handles selfish players well • Can be robust to timing uncertainty	• Overhead to implement punishment • Brittle to hostile users and information uncertainty
Supermodular games	• Many well-known algorithms are supermodular • Extends well to multiple dimensions • Very robust to timing discrepancies	• Weak convergence / stability characteristics with multiple equilibria • Limited in modifications to actions and decision processes
Potential games	• Fairly robust to uncertainty for stability and convergence • Converges in beliefs in learning applications • Lots of flexibility in design of decision processes and actions • Robust to timing discrepancies	• A bit of work to find utility functions that correlate with system objectives • Multi-layer / dimension not well-published
Policy Design	• Eliminate known bad states • Combine with any other approach	• Centralized issues – a priori knowledge, overhead, flexibility
Cooperative Design	• Ensures alignment of operating states with desired states	• Overhead and complexity

Why Apply Game Theory to Cognitive Radio Analysis and Design?

- Applying game theory and game models to the analysis of cognitive radio interactions
  - Provides a natural method for modeling cognitive radio interactions
  - Significantly speeds up and simplifies the analysis process (can be performed at the undergraduate level – Senior EE)
  - Permits analysis without well defined decision processes (only the goals are needed)
  - Can be supplemented with traditional analysis techniques
  - Can provides valuable insights into how to design cognitive radio decision processes
  - Has wide applicability
- Major Applications
  - **Distributed algorithm design**
  - **Jammer vs AJ**
  - Inducing cooperation
  - Market / auction design

It might also be good for your academic career



- Search parameters:
  - http://ieeexplore.ieee.org/search/advsearch.jsp
  - “cognitive radio” AND “game theory” in text or metadata

Summary

- Adaptations of cognitive radios interact
  - Adaptations can have unexpected negative results
    - Infinite recursions, vicious cycles
  - Insufficient to consider behavior of only a single link in the design
- Behavior of collection of radios can be modeled as a game
  - Some differences in models and assumptions but high level mapping is fairly close
- Extend game theory with control theory, machine learning, and related techniques to address other design / analysis objectives
- Many more extensions are possible
  - **Numerous dissertation possibilities**

Potential Games for Distributed Radio Resource Algorithm Design

Subsection Material

- FIP – a very attractive property
- Potential game definition
- Potential game identification

Lesser Rationality: Myopic Processes

- Players have no knowledge about utility functions, or expectations about future play, typically can observe

Definition 4.10: *Best Response Dynamic*
A decision rule $d_i : A \to A_i$ is a best response dynamic if each adaptation would maximize the radio's utility if all other radios continued to implement the same waveforms, i.e., $d_i(a) \in \{b_i \in A_i : u_i(b_i, a_{-i}) \geq u_i(a_i, a_{-i}) \forall a_i \in A_i\}$

Definition 4.11: *Better Response Dynamic*
A decision rule $d_i : A \to A_i$ is a better response dynamic if each adaptation would improve the radio's utility if all other radios continued to implement the same waveforms, i.e., $d_i(a) \in \{b_i \in A_i : u_i(b_i, a_{-i}) > u_i(a_i, a_{-i})\}$.

Definition 4.13: *Friedman's Random Better Response* [Friedman_01]
Player i chooses an action from $A_i \backslash b_i$ where b_i is player i's current action according to a uniform random distribution. If the chosen action would improve the utility of player i, it is implemented, otherwise, the player continues to play b_i.

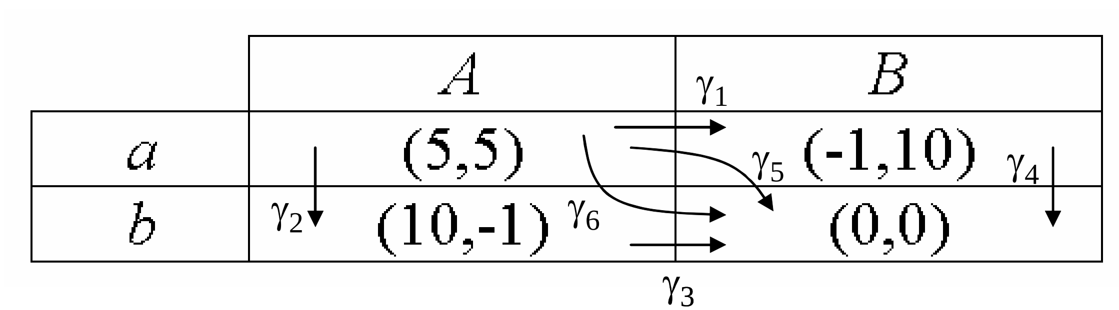
Definition 4.12: *Random Better Response Dynamic* (*)
A decision rule $d_i : A \to A_i$ is a random better response dynamic if for each $t_i \in T_i$, radio i chooses an action from A_i where each action has a nonzero probability of being chosen and implements the action if it would improve its utility.

Paths and Convergence

- *Path* [Monderer_96]
  - A path in Γ is a sequence $\gamma = (a^0, a^1, \ldots)$ such that for every $k \geq 1$ there exists a unique player such that the strategy combinations (a^{k-1}, a^k) differs in exactly one coordinate.
  - Equivalently, a path is a sequence of unilateral deviations. When discussing paths, we make use of the following conventions.
  - Each element of γ is called a *step*.
  - a^0 is referred to as the *initial* or *starting point* of γ.
  - Assuming γ is finite with *m* steps, a^m is called the *terminal point* or *ending point* of γ and say that γ has *length m*.
- *Cycle* [Voorneveld_96]
  - A finite path $\gamma = (a^0, a^1, \ldots, a^k)$ where $a^k = a^0$

Improvement Paths

- Improvement Path
  - A path $\gamma = (a^0, a^1, \ldots)$ where for all $k \geq 1$, $u_i(a^k) > u_i(a^{k-1})$ where i is the unique deviator at k
- Improvement Cycle
  - An improvement path that is also a cycle
  - See the DECT example



$\gamma_1 = ((a, A), (a, B))$	$\gamma_3 = ((b, A), (b, B))$	$\gamma_5 = (\gamma_1, (b, B))$
$\gamma_2 = ((a, A), (b, A))$	$\gamma_4 = ((a, B), (b, B))$	$\gamma_6 = (\gamma_1, (b, B))$

Convergence Properties

- Finite Improvement Property (FIP)
  - All improvement paths in a game are finite
- Weak Finite Improvement Property (weak FIP)
  - From every action tuple, there exists an improvement path that terminates in an NE.
- FIP implies weak FIP
- FIP implies lack of improvement cycles
- Weak FIP implies existence of an NE

FIP Examples

Game with FIP

	A	B
a	1,-1	0,2
b	-1,1	2,2

Weak FIP but not FIP

	A	B	C
a	1,-1	-1,1	0,2
b	-1,1	1,-1	1,2
c	2,0	2,1	2,2

Implications of FIP and weak FIP

- Assumes radios are incapable of reasoning ahead and must react to internal states and current observations
- Unless the game model of a CRN has weak FIP, then no autonomously rational decision rule can be guaranteed to converge from all initial states under random and round-robin timing (Theorem 4.10 in dissertation).
- If the game model of a CRN has FIP, then ALL greedy rational decision rules are guaranteed to converge from all initial states under random and round-robin timing.
  - And asynchronous timings, but not immediate from definition
- More insights possible by considering more refined classes of decision rules and timings

Convergence Results (Finite Games)

	Timings			
Decision Rules	Round-Robin	Random	Synchronous	Asynchronous
Best Response	1,3	1,3	1	1,3
Exhaustive Better Response	3	3	-	3
Random Better Response[(a)]	1,2,3	1,2,3	1,2,3	1,2,3
Random Better Response[(b)]	1,3	1,2,3	1	1,2,3

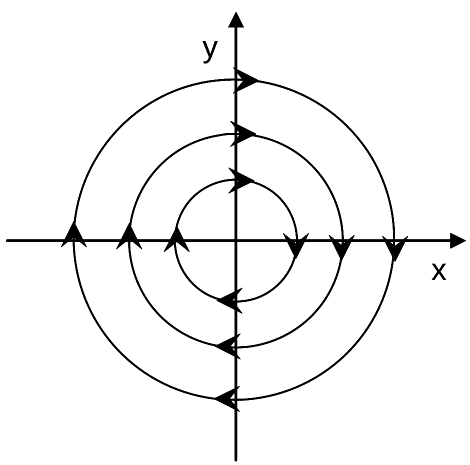
(a) Definition 4.12, (b) Definition 4.13, 1. IESDS, 2. Weak FIP, 3. FIP

- If a decision rule converges under round-robin, random, or synchronous timing, then it also converges under asynchronous timing.
- Random better responses converge for the most decision timings and the most surveyed game conditions.
  - Implies that non-deterministic procedural cognitive radio implementations are a good approach if you don't know much about the network.

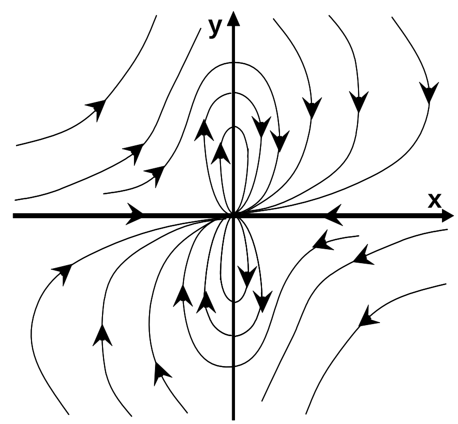
Stability

Definition 3.5: *Lyapunov stability*
We say that an action vector, a^*, is *Lyapunov stable* if for every $\varepsilon > 0$ there is a $\delta > 0$ such that for all $t \geq t^0$, $\left\| a\left(t^0\right), a^* \right\| < \delta \Rightarrow \left\| a(t), a^* \right\| < \varepsilon$ 8.

Definition 3.6: *Attractvity*
The action vector a^* is said to be *attractive* over the region $S \subset A$, $S = \left\{ a \in A \mid \left\| a, a^* \right\| < M \right\}$, if given any $a(t_0) \in S$, the sequence $\left\{ a(t) \right\}$ converges to a^* for $t \geq t_0$.



Stable, but not attractive



Attractive, but not stable

Lyapunov's Direct Method

Theorem 3.3: *Lyapunov's Direct Method for Discrete Time Systems*

Given a recursion $a(t^{k+1}) = d^t(a(t^k))$ *with fixed point* a^**, we know that* a^* *is Lyapunov stable if there exists a continuous function (known as a* **Lyapunov function***) that maps a neighborhood of* a^* *to the real numbers, i.e.,* $L: N(a^*) \rightarrow \mathbb{R}$ *, such that the following three conditions are satisfied:*

1) $L(a^*) = 0$
2) $L(a) > 0 \ \forall a \in N(a^*) \setminus a^*$
3) $\Delta L(a(t)) \equiv L\left[d^t(a(t))\right] - L(a(t)) \leq 0 \quad \forall a \in N(a^*) \setminus a^*$

Further, if conditions 1-3 hold and

a) $N(a^*) = A$*, then* a^* *is globally Lyapunov stable;*
b) $\Delta L(a(t)) < 0 \ \forall a \in N(a^*) \setminus a^*$*, then* a^* *is asymptotically stable;*
c) $N(a^*) = A$ *and* $\Delta L(a(t)) < 0 \ \forall a \in N(a^*) \setminus a^*$*, then* a^* *is globally asymptotically stable.*

Left unanswered: where does *L* come from?
Can it be inferred from radio goals?

Potential Games

Potential Game Content

- Define / Introduce Potential Games
- Key Properties
- Identification / Design
- Examples

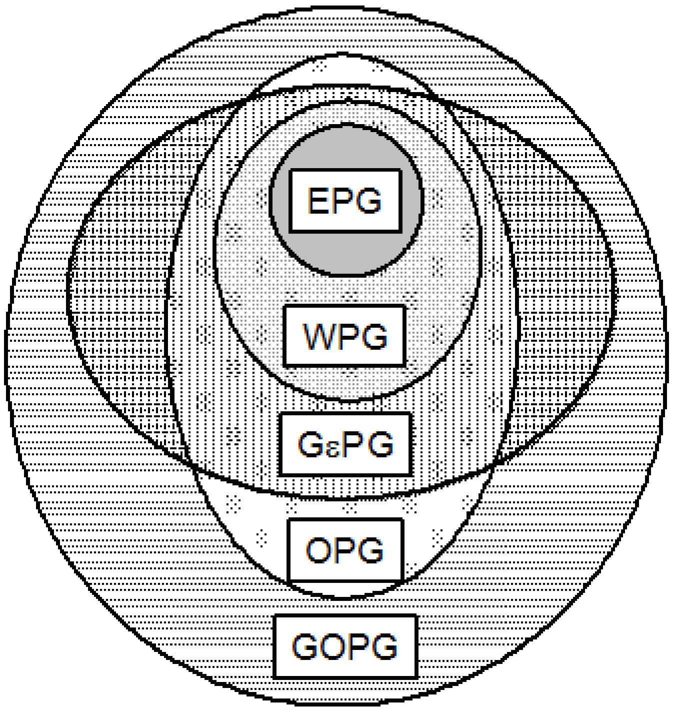
Wish List for a Distributed CR Network

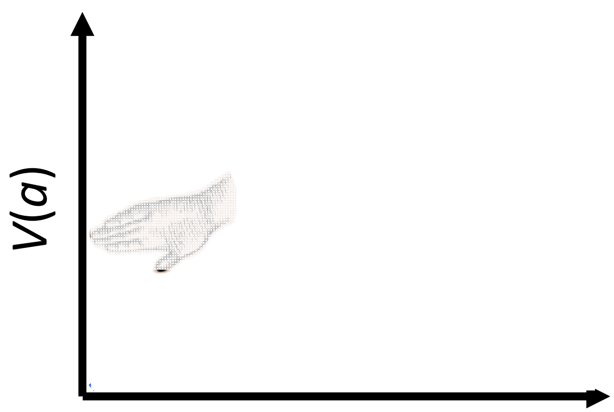
- Predictable behavior
- Broad range of convergence and stability conditions
  - Myopic
    - Only has to be able to evaluate impact of own adaptaitons
  - Ideally decouple goal selection from decision rules
- Minimal communications
  - Looking for emergent behavior that coincides with design objective
- A lot of flexibility for individual radio customization

- **All features of CR networks based on potential games**

Potential Games

- Existence of a function (called the potential function, *V*), that reflects the change in utility seen by a unilaterally deviating player.
- Cognitive radio interpretation:
  - Every time a cognitive radio unilaterally adapts in a way that furthers its own goal, some real-valued function increases.

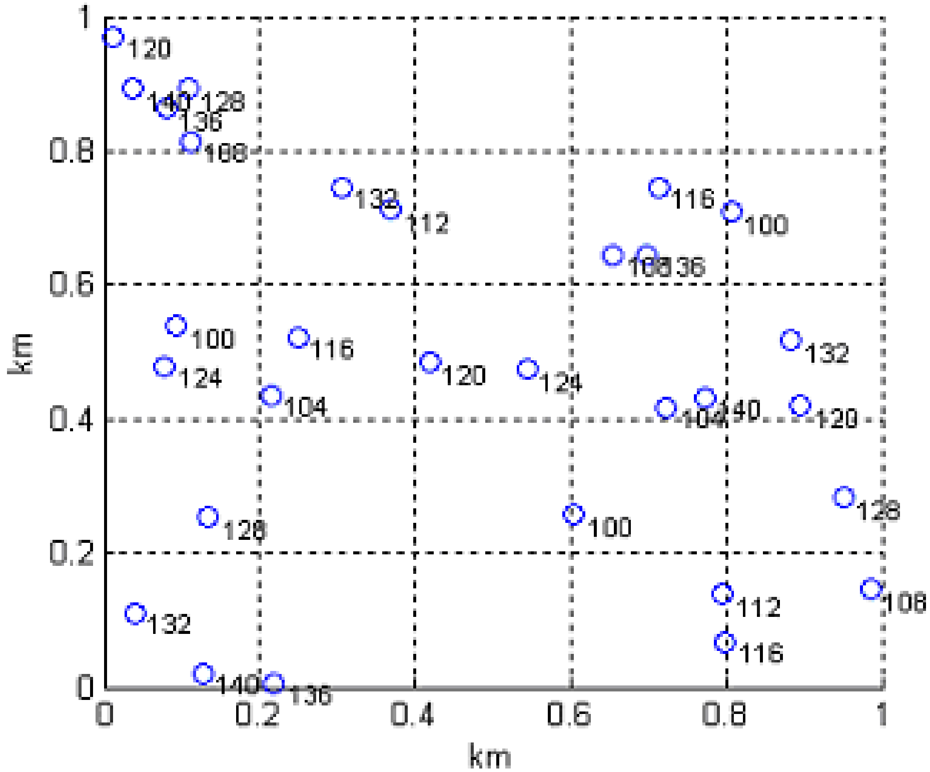


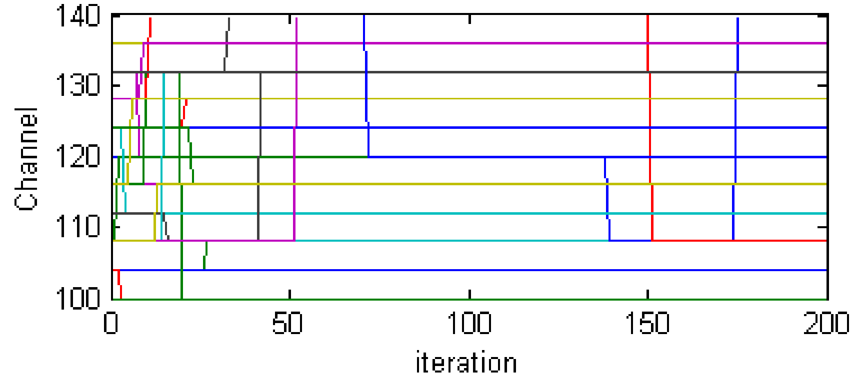


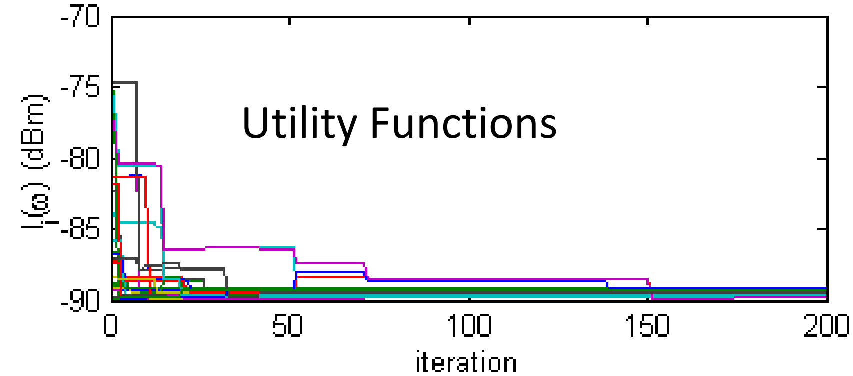
Potential Game	Relationship ($\forall i \in N, \forall a \in A$)
Exact (EPG)	$u_i(b_i,a_{-i}) - u_i(a_i,a_{-i}) = V(b_i,a_{-i}) - V(a_i,a_{-i})$
Weighted (WPG)	$u_i(b_i,a_{-i}) - u_i(a_i,a_{-i}) = \alpha_i\left[V(b_i,a_{-i}) - V(a_i,a_{-i})\right]$
Ordinal (OPG)	$u_i(b_i,a_{-i}) - u_i(a_i,a_{-i}) > 0 \Leftrightarrow V(b_i,a_{-i}) - V(a_i,a_{-i}) > 0$
Generalized Ordinal (GOPG)	$u_i(b_i,a_{-i}) - u_i(a_i,a_{-i}) > 0 \Rightarrow V(b_i,a_{-i}) - V(a_i,a_{-i}) > 0$
Generalized ε (GεPG)	$u_i(b_i,a_{-i}) > u_i(a_i,a_{-i}) + \varepsilon_1 \Rightarrow V(b_i,a_{-i}) > V(a_i,a_{-i}) + \varepsilon_2$

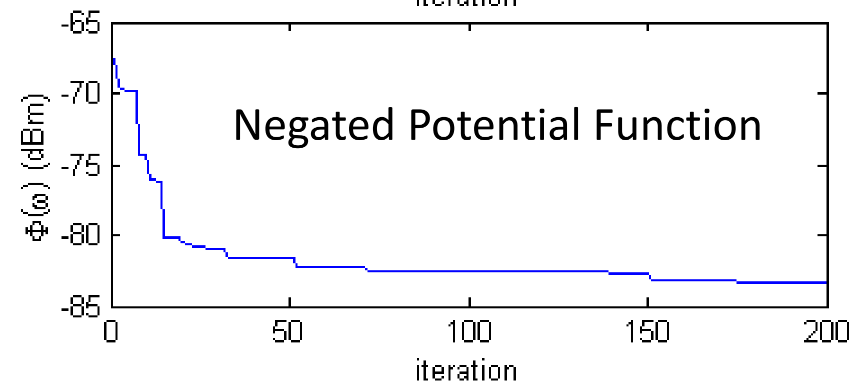
A Dynamic Frequency Selection Algorithm

Final channels by access node









Implications of Monotonicity

- Monotonicity implies
  - Existence of steady-states (maximizers of *V*)
  - Convergence to maximizers of *V* for numerous combinations of decision timings decision rules – all self-interested adaptations
- Does not mean that that we get good performance
  - Only if *V* is a function we want to maximize

	Timings			
Decision Rules	Round-Robin	Random	Synchronous	Asynchronous
Best Response	1,2,4	1,2,4	-	1,2
Exhaustive Better Response	1,2	1,2	-	1,2
Random Better Response[(a)]	1,2,4	1,2,4	1,2	1,2
Random Better Response[(b)]	1,2	1,2	-	1,2
ε-Better Response[(c)]	1,2,3,4	1,2,3,4	-	1,2,3
Intelligently Random Better Response	1,4	1,4	-	1,2
Directional Better Response[(c)]	4	4	-	-
Averaged Best Response[(d)]	3,4	3,4	-	-

(a) Definition 4.12, (b) Definition 4.13, (c) Convergence to an ε-NE, (d) u_i quasi-concave in a_i

1.Finite game, 2. Infinite game with FIP, 3. Infinite game with AFIP, 4. Infinite game with bounded continuous potential function (implication of D^V)

Exact Potential Games Form a Linear Space

Definition 5.15: *Linear Space*
Given a set X, X is said to be a *linear space* if for every $x, y, z \in X$ and every $\alpha, \beta \in \mathbb{R}$ it satisfies the following ten (10) properties:

(1) Closure under addition, $x + y \in X$
(2) Closure under scalar multiplication, i.e., $\alpha x \in X$
(3) Commutativity, i.e., $x + y = y + x$
(4) Additive Associativity, i.e., $x + y = y + x$
(5) Additive Identity, i.e., there is some $0 \in X$ such that if $x \in X$, $0 + x = X$.
(6) Additive Inverse, i.e., for every $x \in X$, there is some $-x \in X$ such that $x + (-x) = 0$.
(7) Associativity of Scalar Multiplication, i.e., $\alpha(\beta x) = (\alpha\beta)x$
(8) Distributivity of Scalar Sums, i.e., $(\alpha + \beta)x = \alpha x + \beta x$
(9) Distributivity of Vector Sums, i.e., $\alpha(x + y) = \alpha x + \alpha y$
(10) Scalar Multiplicative Identity, i.e., $1x=x$.

Theorem 5.24: *Linear Space of Exact Potential Games* [Fachini_97]
$\Gamma^{N,A}$ forms a linear space
Proof: A proof of this result is given in [Fachini_97]. However, some key aspects of this proof are repeated in the following. An additive identity element is given by the game $\Gamma = \langle N, A, \{0\} \rangle$ which has exact potential function $V(a) = 0$. Given exact potential games, $\Gamma_1, \Gamma_2 \in \Gamma^{N,A}$ with potential functions V_1 and V_2, and scalars $\alpha_1, \alpha_2 \in \mathbb{R}$, $\Gamma_3 = \alpha_1\Gamma_1 + \alpha_2\Gamma_2$, then Γ_3 is an exact potential game with potential $V_3 = \alpha_1 V_1 + \alpha_2 V_2$.□

Potential Game Properties Related to Network Behavior

- All finite potential games have FIP
- All finite games with FIP are (Generalized Ordinal) potential games
- -*V* is a Lyapunov function for isolated maximizers
- Stable NE solvable by maximizers of *V*
- Maximizer of potential game need not maximize your objective function
  - Cognitive Radios' Dilemma is a potential game
- Items which I'm not really covering today
  - Potential games have the Fictitious Play Property
  - Connected to Shapley value

Exact Potential Game Forms

- Many exact potential games can be … the form of th… …ion

Can't Influence Own Outcome

Network-wide Objective Function

Only impacted by self

Sum of mini-coordination games

Game		…ion
Coordination Game	…)	$V(a) = C(a)$
Dummy Game	… (a) … $D_i(a_{-i})$	$V(a) = c, c \in \mathbb{R}$
Coordinati… Game	… $(a) + D_i(a_{-i})$	$V(a) = C(a)$
Self-Motivated Game	$u_i(a) = S_i(a_i)$	… $\sum S_i(a_i)$
Bilateral Symmetric Interaction (BSI) Game	$u_i(a) = \sum_{j \in N \setminus \{i\}} w_{ij}(a_i, a_j) - \ldots$ where $w_{ij}(a_i, a_j) = w_{ji}(a_j, a_i)$	… $-\sum_{i \in N} S_i(a_i)$
Multilateral Symmetric Interaction (MSI) Game	$u_i(a) = \sum_{\{S \in 2^N : i \in S\}} w_{S,i}(a_S) + D_i(a_{-i})$ where $w_{S,i}(a_S) = w_{S,j}(a_S) \forall i, j \in S$	$V(a) = \sum_{S \in 2^N} w_S(a_S)$

Congestion Games (1/2)

- Key components
  - a finite set of actors (players) $N=\{1,2,\ldots,n\}$
  - a set of facilities, $F=\{1, 2, \ldots, g\}$
  - a set of payoffs, $c_f(k)$ where and k is the number of users of facility f.
- Game Model
  - $N = N$
  - $A_i = 2^F$ (choose any subset of F)

$$u_i(a) = \sum_{f \in a_i} c_f\left(\sigma_f(a)\right)$$

  - Sum of payoffs of each facility

$$\sigma_f(a) = \#\left\{i \in N : f \in a_i\right\}$$

  - Each facility has its own function, function of # of users (anonymous)

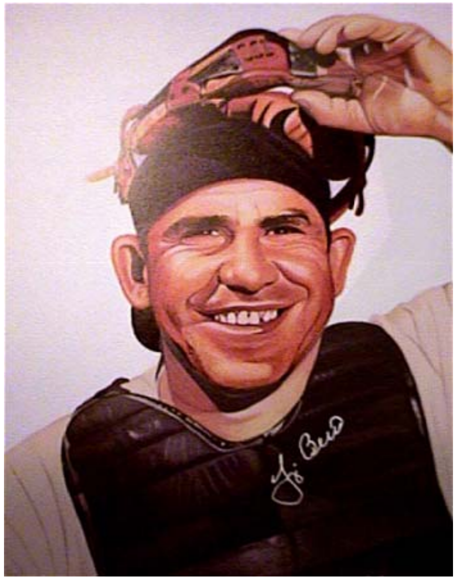
Congestion Games (2/2)

- Exact potential function
  - Sum of facility costs over all used facilities from 1 to # of users of each facility

$$V(a) = \sum_{f \in \bigcup_{i=1}^{n} a_i} \left(\sum_{k=1}^{\sigma_f(a)} c_f(k) \right)$$

- Comments:
  - Every player does not need the same action set for EPG to hold
  - Tends to either spread (costly) or concentrate (beneficial) users across facilities (as modified by club benefits)
- Examples:
  - Routers (though not with prioritization)
  - Vehicle Traffic congestion
  - Some MACs
  - Could be network selection

"Nobody goes there anymore. It's too crowded"
-Yogi Berra



(actually John McNulty)

Other Exact Potential Game Identification Techniques

- Linear Combination of Exact Potential Game Forms [Fachini_97]
  - If <N,A,{u_i}> and <N,A,{v_i}> are EPG, then <N,A,{$\alpha u_i + \beta v_i$}> is an EPG
  - Does not hold for other forms of potential games
- Evaluation of second order derivative [Monderer_96]

$$\frac{\partial^2 u_i}{\partial a_i \partial a_j} = \frac{\partial^2 u_j}{\partial a_i \partial a_j}, \forall i \neq j \in N, a \in A$$

Ordinal Potential Game Identification

- Lack of weak improvement cycles [Voorneveld_97]
- FIP and no action tuples such that

$$u_i(a_i, a_{-i}) = u_i(b_i, a_{-i}), b_i \neq a_i$$

- Better response equivalence to an exact potential game [Neel_04]

Not an OPG

Γ_1	A	B
a	(1,0)	(2,0)
b	(2,0)	(0,1)

An OPG

Γ_2	A	B
A	(1,-1)	(2,0)
B	(2,0)	(0,1)

Definition 5.9: *Better-response equivalence*

A game $\Gamma = \langle N, A, \{u_i\} \rangle$ is said to be *better response equivalent* to game $\Gamma' = \langle N, A, \{v_i\} \rangle$ if $\forall i \in N$, $a \in A$, $u_i(a_i, a_{-i}) > u_i(b_i, a_{-i}) \Leftrightarrow v_i(a_i, a_{-i}) > v_i(b_i, a_{-i})$.

Example Identification

- Single cluster target SINR

$$u_i(\mathbf{p}) = -\left|\hat{\gamma} - \frac{g_i p_i}{1/K\left(\sum_{k\in N\setminus i} g_k p_k + \sigma\right)}\right|$$

- Better Response Equivalent

$$u_i'(\mathbf{p}) = -\left[\hat{\gamma}/K\left(\sum_{k\in N\setminus i} g_k p_k + \sigma\right) - g_i p_i\right]^2$$

$$u_i'(\mathbf{p}) = -g_i^2 p_i^2 + 2\hat{\gamma}/K\sigma g_i p_i$$

Self-motivated game

$$+2\hat{\gamma}/K\left(\sum_{k\in N\setminus i} g_i g_k p_i p_k\right)$$

BSI game

$$-\left[\hat{\gamma}/K\left(\sum_{k\in N\setminus i} g_k p_k + \sigma\right)\right]^2$$

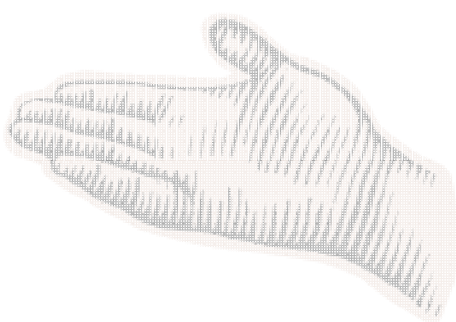
Dummy game

$$V(\mathbf{p}) = 2\hat{\gamma}/K\left(\sum_{i\in N}\sum_{i>k}^{n} g_i g_k p_i p_k\right) + \sum_{i\in N}\left(-g_i^2 p_i^2 + 2\hat{\gamma}/K\sigma g_i p_i\right)$$

Ordinal Potential Game Comments

- Obviously applies to a broader class of algorithms (every EPG is an OPG)
- Doesn’t generally have the linearity properties needed for linear combinations (additive)

Interference Reducing Networks



"…he intends only his own gain, and he is in this, as in many other cases, **led by an invisible hand** to promote an end which was no part of his intention. Nor is it always the worse for society that it was no part of his intention. By pursuing his own interest he frequently promotes that of the society more effectually than when he really intends to promote it." – A. Smith, *Wealth of Nations*

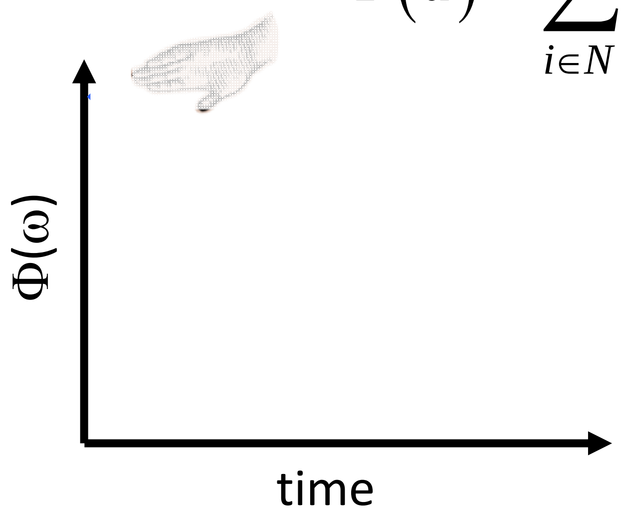
Using Potential Games to Design of Cognitive Radio Networks

- If we design our networks to be an exact potential games, then we can
  - Predict steady-state behavior (maximizers of *V)*
  - Know that very simple greedy algorithms will converge
  - Know that very simple algorithms will be stable
  - Scale up more complex algorithms
- Issues:
  - Potential function should be something we want maximized
  - Stability only holds for isolated fixed points
  - Minimize amount of external information / information exchange
- Approach
  - Find objectives that look like exact potential game utility functions that correspond
  - Look for local ways to gather information
    - Trivial to make desirable exact potential game out of coordination games
    - Possible to use other forms, may require modifying observations or defining specific network processes

Basic Model

- Each radio (link, network) attempts to minimize its own interference function, $I_i : \Omega \rightarrow \mathbb{R}$, by adapting its waveform ω_l according to its decision rule $d_i : \Omega \rightarrow \Omega_i$
- Network $\langle N, A, \{u_i\}, \{d_i\}, T \rangle$ is said to be an interference reducing network if $\Phi(d_i(\omega)) \leq \Phi(\omega)$ for all $\omega \in \Omega$ and all $i \in N$ where

$$\Phi(a) = \sum_{i \in N} I_i(a)$$



Connection with Potential Games

- Definition of IRN can be satisfied if

$$I_i\left(\omega_i^*,\omega_{-i}\right) < I_i\left(\omega_i,\omega_{-i}\right) \Rightarrow \Phi\left(\omega_i^*,\omega_{-i}\right) < \Phi\left(\omega_i,\omega_{-i}\right)$$

- Logically if $u_i(\omega) = -I_i(\omega)$, then

$$I_i\left(\omega_i^*,\omega_{-i}\right) < u_i\left(\omega_i,\omega_{-i}\right) \Rightarrow u_i\left(\omega_i^*,\omega_{-i}\right) > u_i\left(\omega_i,\omega_{-i}\right)$$

- So $V = -\Phi$ is a GOPF for this network
- In other words, IRN can be realized by designing CRN as a potential game with $V(\omega) \propto -\Phi(\omega)$

Designing IRN as a Potential Game

- Recall that easier to initially design an exact potential game than an ordinal potential game
- Approach: Try to fit IRN into one of the known exact potential game forms

Game	Utility Function Form	Potential Function
Coordination Game	$u_i(a) = C(a)$	$V(a) = C(a)$
Dummy Game	$u_i(a) = D_i(a_{-i})$	$V(a) = c, c \in \mathbb{R}$
Coordination-Dummy Game	$u_i(a) = C(a) + D_i(a_{-i})$	$V(a) = C(a)$
Self-Motivated Game	$u_i(a) = S_i(a_i)$	$V(a) = \sum_{i \in N} S_i(a_i)$
Bilateral Symmetric Interaction (BSI) Game	$u_i(a) = \sum_{j \in N \setminus \{i\}} w_{ij}(a_i, a_j) - S_i(a_i)$ where $w_{ij}(a_i, a_j) = w_{ji}(a_j, a_i)$	$V(a) = \sum_{i \in N} \sum_{j=1}^{i-1} w_{ij}(a_i, a_j) - \sum_{i \in N} S_i(a_i)$
Multilateral Symmetric Interaction (MSI) Game	$u_i(a) = \sum_{\{S \in 2^N : i \in S\}} w_{S,i}(a_S) + D_i(a_{-i})$ where $w_{S,i}(a_S) = w_{S,j}(a_S) \forall i, j \in S$	$V(a) = \sum_{S \in 2^N} w_S(a_S)$

Mapping Exact Potential Games to IRN Types

	Game	Utility Function Form	Potential Function
Altruism	Coordination Game	$u_i(a) = C(a)$	$V(a) = C(a)$
	Dummy Game	$u_i(a) = D_i(a_{-i})$	$V(a) = c, c \in \mathbb{R}$
	Coordination-Dummy Game	$u_i(a) = C(a) + D_i(a_{-i})$	$V(a) = C(a)$
Isolated	Self-Motivated Game	$u_i(a) = S_i(a_i)$	$V(a) = \sum_{i \in N} S_i(a_i)$
Bilateral Symmetric Interference	Bilateral Symmetric Interaction (BSI) Game	$u_i(a) = \sum_{j \in N \setminus \{i\}} w_{ij}(a_i, a_j) - S_i(a_i)$ where $w_{ij}(a_i, a_j) = w_{ji}(a_j, a_i)$	$V(a) = \sum_{i \in N} \sum_{j=1}^{i-1} w_{ij}(a_i, a_j) - \sum_{i \in N} S_i(a_i)$
Coalitions / Logical Node?	Multilateral Symmetric Interaction (MSI) Game	$u_i(a) = \sum_{\{S \in 2^N : i \in S\}} w_{S,i}(a_S) + D_i(a_{-i})$ where $w_{S,i}(a_S) = w_{S,j}(a_S) \forall i, j \in S$	$V(a) = \sum_{S \in 2^N} w_S(a_S)$

Self-Motivated Parameters

- Internal device settings
  - Sampling rate, filters, RFIC parameters…
- Non-interactive parameters
  - Modulation (same PSD for BPSK, QPSK, QAM)
  - Error Correction
  - Interleaving
  - Others…
- Goals which drive those can be folded into any of the preceding algorithms as games of self-interested parameters are exact potential games and thus part of the linear space

$$\tilde{u}_i(a) = u_{i,1}(f, p, \ldots) + u_{i,2}(a_i)$$

- More complicated when there's interactions

Globally Altruistic Networks (Explicit Information)

- Radio goal: minimize network interference

$$u_i(\omega) = -\sum_{k \in N} \sum_{j \in N \backslash k} I_i(\omega)$$

- Potential, Interference Function

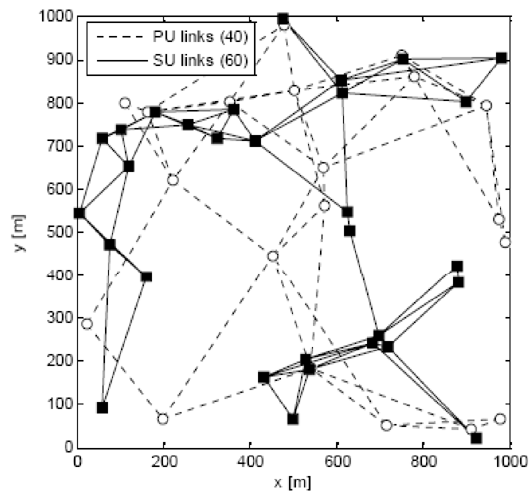
$$\Phi(\omega) = -V(\omega) = \sum_{k \in N} \sum_{j \in N \backslash k} I_i(\omega)$$

- Unique benefit: Works for all waveform adaptations
- Unique drawback: Lots of overhead – may need functional radio environment map
- Proposed algorithms that satisfy GAN: [Sung], [Nie]

Example Application:

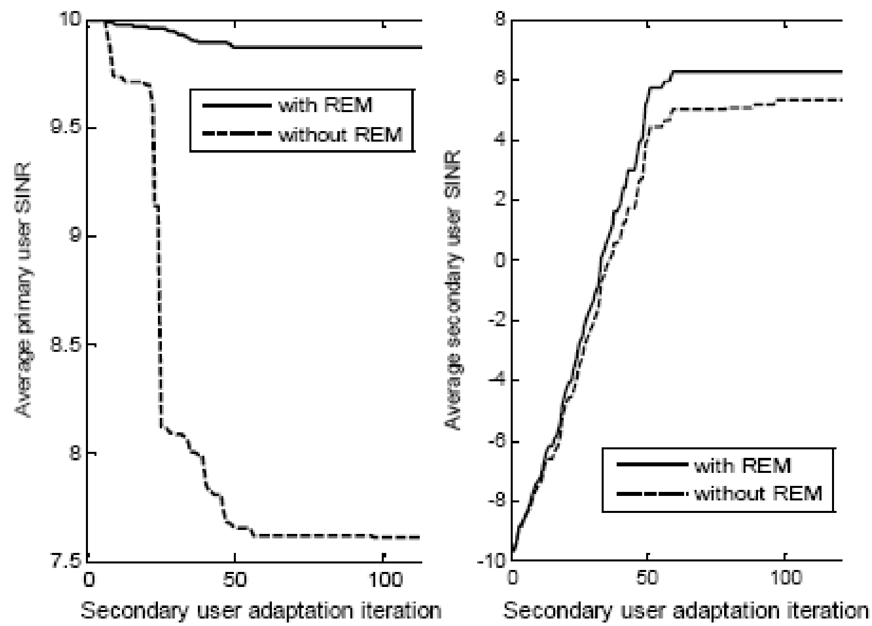
- Overlay network of secondary users (SU) free to adapt power, transmit time, and channel

- Without REM:
  - Decisions solely based on link SINR
- With REM
  - Radios effectively know everything



Parameter	Value
Transmission range of radio node (PU or SU)	450 meters
Sensing range of SU	450 meters
Interference range of SU	450 meters
Speed of SUs	Uniformly distributed in (0, 10m/s)
Data rate of wireless link	2 Mbps
Interface queue length	50 packets
Radio channel model	two-ray ground model
Simulation period	200 seconds

Upshot: A little gain for the secondary users; big gain for primary users



From: Y. Zhao, J. Gaeddert, K. Bae, J. Reed, "Radio Environment Map Enabled Situation-Aware Cognitive Radio Learning Algorithms," SDR Forum Technical Conference 2006.

Locally Altruistic Networks (Explicit Information)

- Let $\mathcal{I}_i \subseteq N$ denote the set of radios which are close enough that *i* produces non-negligible interference.
- Goal: minimize interference of those within "range"

$$u_i(\omega) = -\sum_{k \in \mathcal{I}_i} \sum_{j \in \mathcal{I}_i \setminus k} I_i(\omega)$$

- Same interference and potential function as before (just eliminated terms for which $I_i = 0$)
- Benefit – Less overhead, just as generalizable
- Drawback – Need extra routine to identify $\mathcal{I}_i$

Isolated Adaptations

- Concept: If an adaptation does not impact the performance of other radios and sum interference is a monotonically decreasing function, then network is said to be an *isolated adaptation* network
- Successful implementation is very much dependent on the radios' action set
- Allowable adaptations:
  - Modulation (assuming PSD doesn't change), interleaving, error correction, receive beamforming, internal settings (e.g., sampling rates, AGC gains. receive filters)

Bilateral Symmetric Interference

- Two cognitive radios, $j,k \in N$, exhibit *bilateral symmetric interference* if

$$g_{jk} p_j \rho\left(\omega_j, \omega_k\right) = g_{kj} p_k \rho\left(\omega_k, \omega_j\right) \quad \forall \omega_j \in \Omega_j, \forall \omega_k \in \Omega_k$$

- ω_k – waveform of radio *k*
- p_k - the transmission power of radio *k*'s waveform
- g_{kj} - link gain from the transmission source of radio *k*'s signal to the point where radio *j* measures its interference,
- $\rho\left(\omega_k, \omega_j\right)$ - the fraction of radio *k*'s signal that radio *j* cannot exclude via processing (perhaps via filtering, despreading, or MUD techniques).

What's good for the goose, is good for the gander…

Source: http://radio.weblogs.com/0120124/Graphics/geese2.jpg

Proof:

- By bilateral symmetric interference

  $$g_{ki} p_k \rho(\omega_k, \omega_i) = g_{ik} p_i \rho(\omega_i, \omega_k) = b_{ik}(\omega_i, \omega_k)$$

- Rewrite goal

  $$u_i(\omega) = -\sum_{k \in N \setminus i} b_{ik}(\omega_i, \omega_k)$$

- Therefore a BSI game (S_i =0) (an EPG)

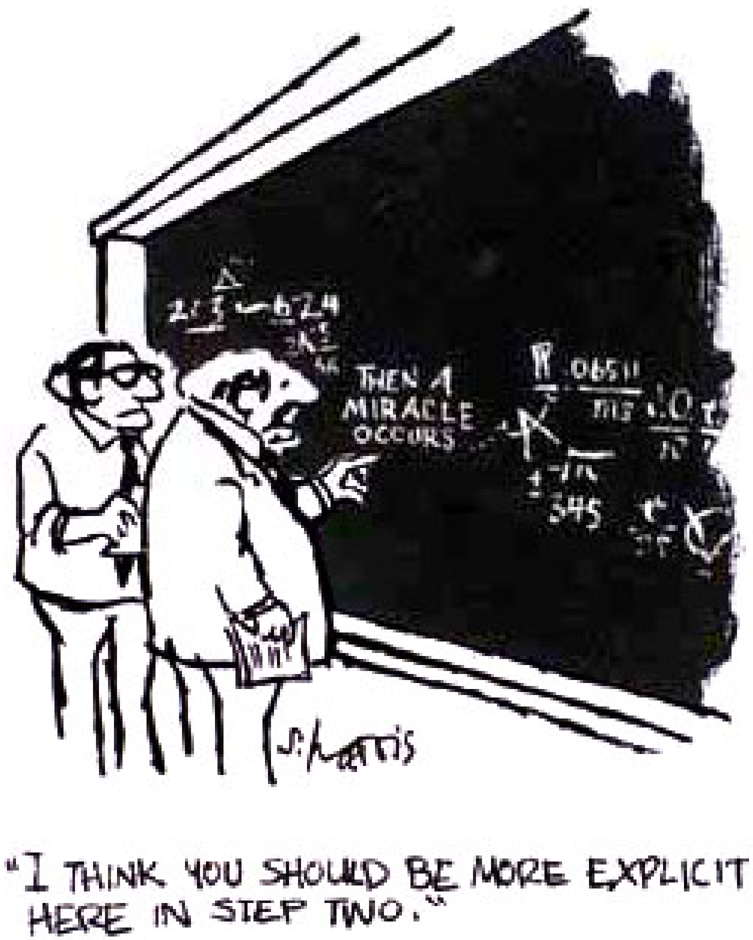
  $$V(\omega) = -\sum_{i \in N} \sum_{k=1}^{i-1} g_{ki} p_k \rho(\omega_k, \omega_i)$$

- Interference Function

  $$\Phi(\omega) = -2V(\omega)$$

- Therefore unilateral deviations increase *V* and decrease $\Phi(\omega)$ – an IRN

Situations where BSI occurs



- Isolated Network Clusters
  - All devices communicate with a common access node with identical received powers.
  - Clusters are isolated in signal space
- Close Proximity Networks
  - All devices are sufficiently close enough that waveform correlation effects dominate
- Controlled Observation Processes
  - Leverage knowledge of waveform protocol to control observations to achieve BSI

Coalitional Interference Reducing Networks

- Concept: Suppose a device is simultaneously a member of several different coalitions or logical nodes and it measures its performance as a sum over its coalitions

$$u_i(a) = \sum_{\{S \subseteq N : i \in S\}} w_{S,i}(a_S)$$

- Could be addressed in any of preceding forms, though altruism within coalition may make most sense
- Non-obvious insight is that summing across coalitions will work

General Comments on Implementing IRNs

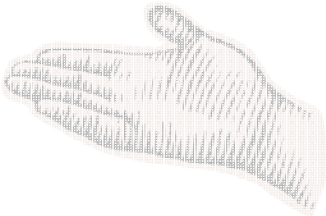
<u>Explicit information</u>

- Gather interference information from other devices in the network
- Conceptually obvious implementations
- Scales badly
- A "bureaucratic nightmare"

Image source: http://unbeknownst.net/images/bureaucracy.jpg

<u>Implicit information</u>

- Design network such that adaptations implement an IRN without gathering information on other devices' interference
- Scales well – ideal solution
- Non-obvious how to implement
  - Invisible hand of cognitive radio?



Isolated Network Clusters

- In this operational scenario, the network consists of a set of clusters *C* for which the following operational assumptions hold:
  - Perhaps through judicious frequency or code reuse between clusters, each radio *i* is operating in a cluster $c \in C$ for which $\mathfrak{I}_i$ is a subset of the cluster.
  - The cluster head enforces a uniform receive power, r_c, on all radios *k* for signals transmitted to the cluster head.
  - Waveforms are restricted to those waveforms for which

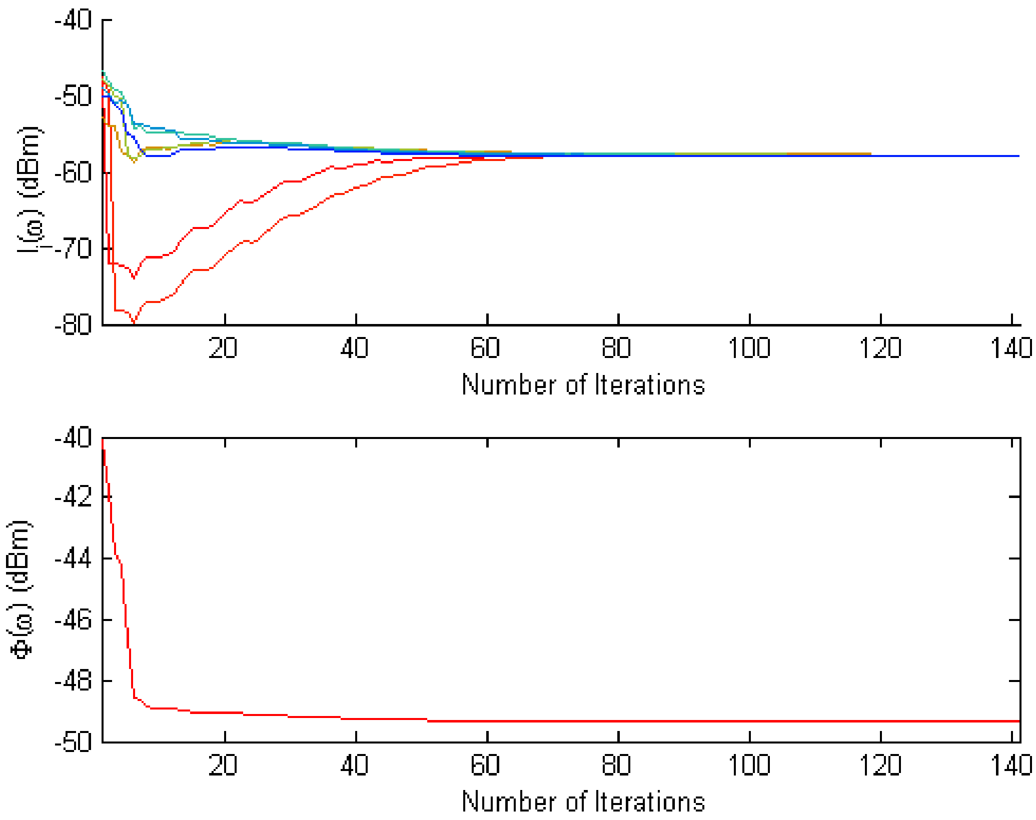
$$\rho(\omega_k, \omega_i) = \rho(\omega_i, \omega_k)$$

  - Cluster heads provide interference measurements to all client radios in the cluster.
- Therefore

$$g_{jk} p_j \rho(\omega_j, \omega_k) = g_{kj} p_k \rho(\omega_k, \omega_j)$$

Example Simulation

- Single cell
- 7 cognitive radios
- 6 code dimensions
- Interference minimizing
- Round-robin



Close Proximity Networks

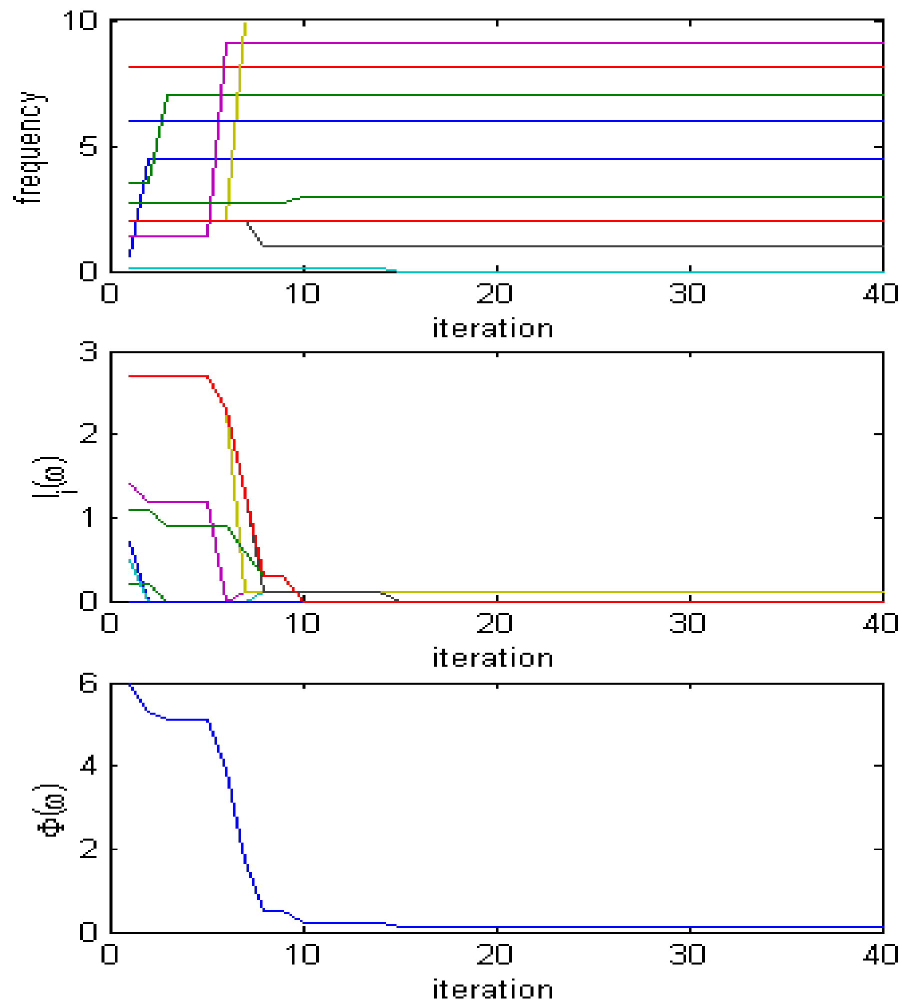
- In this operational scenario it is assumed that the radios are operating as an ad-hoc network in sufficiently close proximity and transmitting with sufficiently similar power levels that waveform correlation dominates the distance and transmitted power effects are negligible. Under these assumptions $-I_i$ is equivalent to
- Further, assume

$$u_i(\omega) = -\sum_{k \in N \setminus i} \rho(\omega_k, \omega_i)$$

$$\rho(\omega_k, \omega_i) = \rho(\omega_i, \omega_k)$$

DFS Close Proximity Network Simulation

- Specific parameters
  - Signal bandwidth = 1 MHz
  - Channel bandwidth = 10 Mhz
  - 10 (decision making) links
  - Frequency discretized with center frequencies every 0.1 MHz
  - Random initial frequencies



Controlled Observation Process

- Concept:
  - Control the radios' observation processes so that they only observe signals where

$$g_{jk} p_j \rho\left(\omega_j, \omega_k\right) = g_{kj} p_k \rho\left(\omega_k, \omega_j\right)$$

- Is this possible to do with meaningful results?

An IRN DFS Algorithm

- Suppose each **access node measures** the received signal **power** and **frequency** of the RTS/CTS sent by other access nodes in the network.
- Assumed out-of-channel interference is negligible and RTS/CTS transmitted at same power
- Then

$$\rho(\omega_k,\omega_i)=\rho(\omega_i,\omega_k)$$
$$\forall\omega_j\in\Omega_j,\forall\omega_k\in\Omega_k$$

$$g_{jk}p_j\rho(\omega_j,\omega_k)=g_{kj}p_k\rho(\omega_k,\omega_j)$$

- Example: 802.11h DFS implemented at access nodes with interference measurements only at access nodes

$$u_i(f)=-I_i(f)=-\sum_{k\in N\setminus i}g_{ki}p_k\sigma(f_i,f_k)$$

$$\sigma(f_i,f_k)=\begin{cases}1 & f_i=f_k\\ 0 & f_i\neq f_k\end{cases}$$

$$\Phi(f)=\sum_{i\in N}\sum_{k\in N\setminus i}g_{ki}p_k\sigma(f_k,f_i)$$

- Exact Potential V=-Φ/2

  => Convergence to a sum interference minimizer from all starting points for all non-synchronous self-interested decision rules

A DFS simulation of the process

- 30 cognitive access nodes
- Upper 5 GHz 802.11 band
- Choose channel with lowest interference
- One randomly selected access node adapts at each instance
- *n*=3
- Random initial channels
- Randomly distributed positions over 1 km^2
- Random timing
- n=3 path loss exponent

$$I_i(f) = \sum_{k \in N \setminus i} p g_{ki} \rho(f_i, f_k)$$

$$\rho(f_i, f_k) = \begin{cases} 1 & f_i = f_k \\ 0 & otherwise \end{cases}$$

- Game is an exact potential game

$$\Phi(f) = p \sum_{j=1}^{30} \sum_{k=i+1}^{30} g_{jk} \rho(f_j, f_k)$$

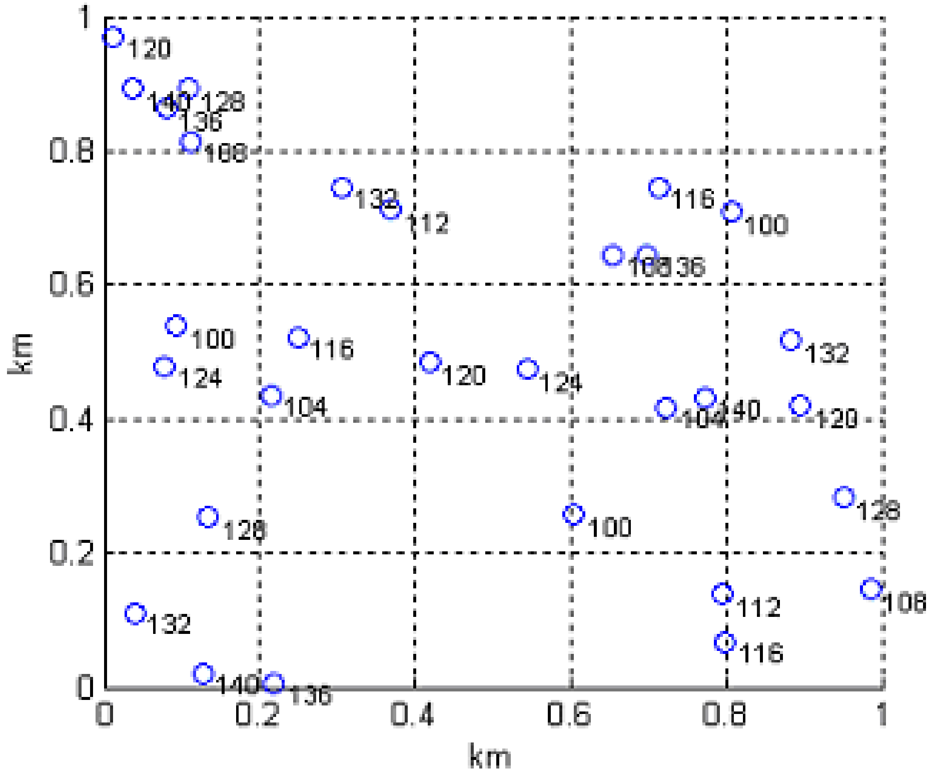
$$V(f) = -\Phi(f)$$

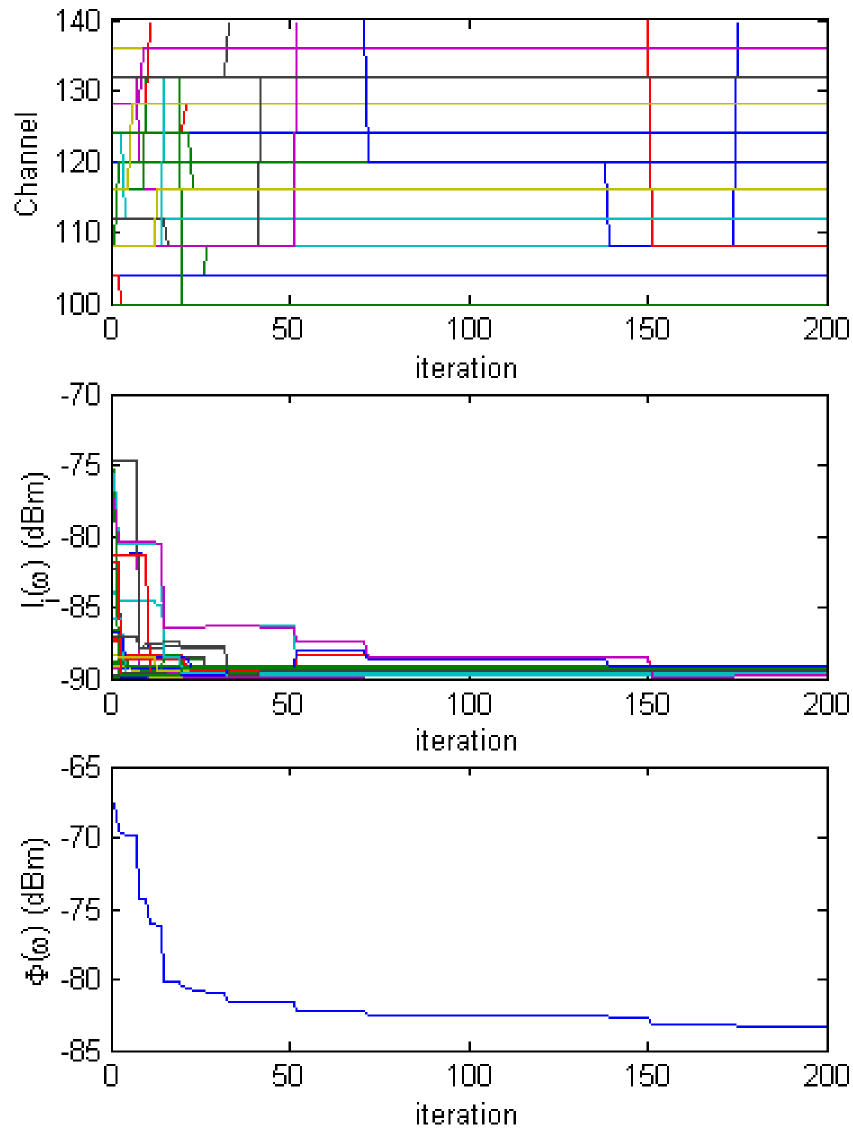
What do we expect based on analysis?

- Steady-states
  - Low interference states
- Convergence
  - Timings (not synchronous)
    - Round-robin, Random, and Asynchronous timing
  - Decision rules (All better response)
    - Best response, random better responses, ε-better response
- Stability
  - Isolated equilibria are stable
- Can add the utility functions of any exact potential game (same action space) and preserve convergence properties though steady-states may change

Dynamic Frequency Selection

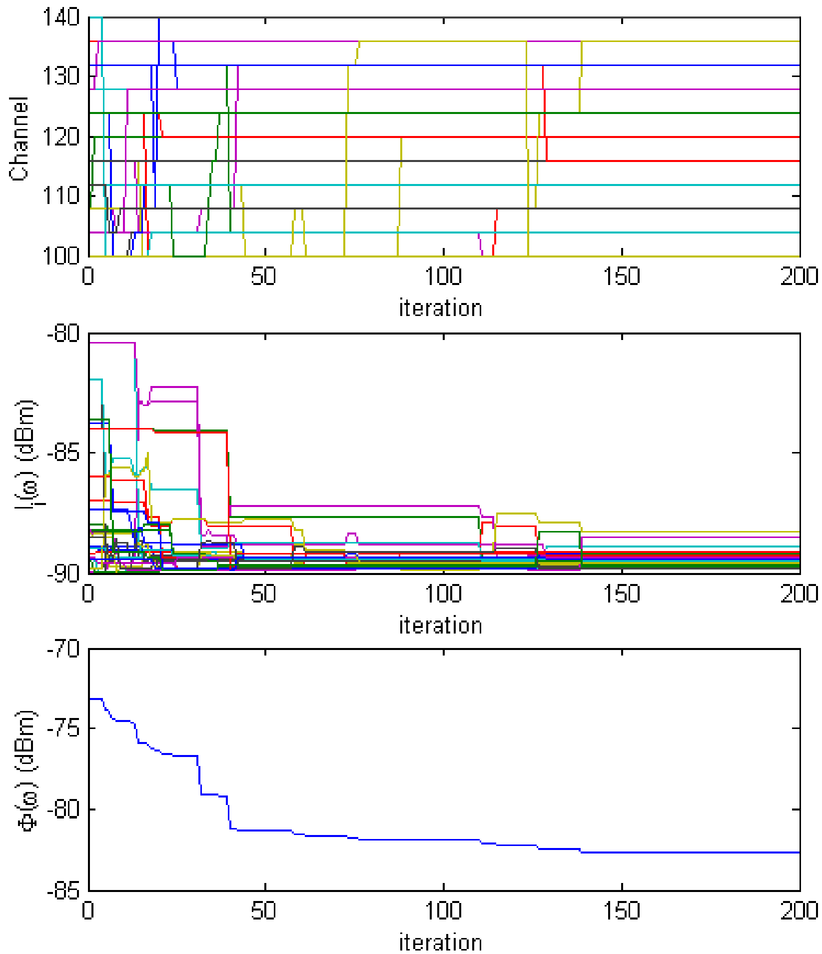
Final channels by access node



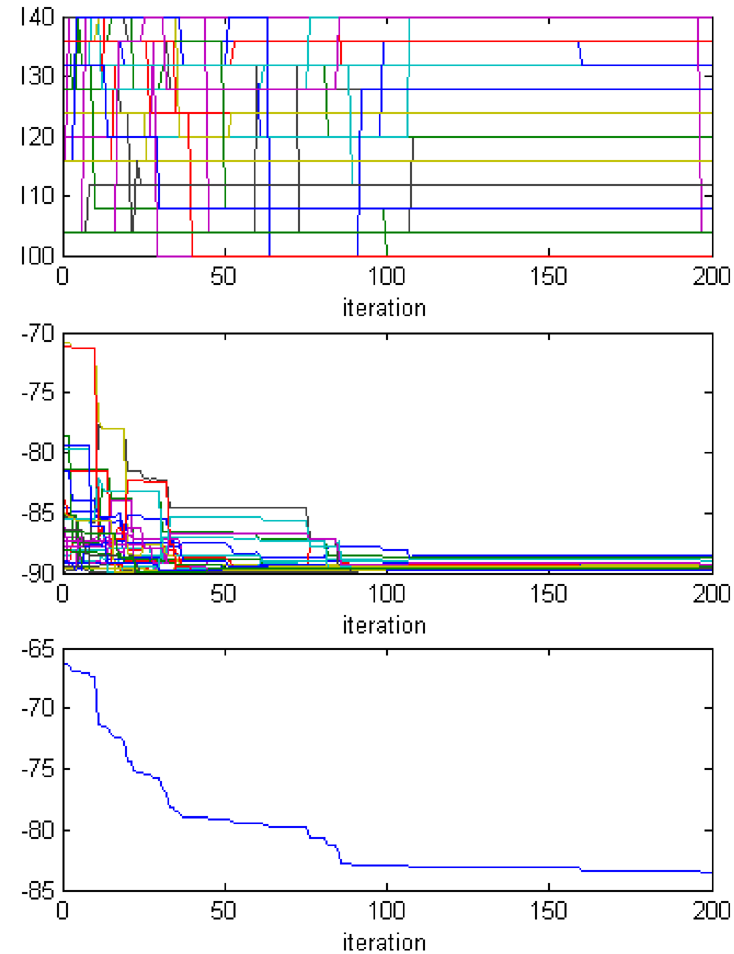


Noiseless suboptimal adaptations

(lowest frequency that improves)

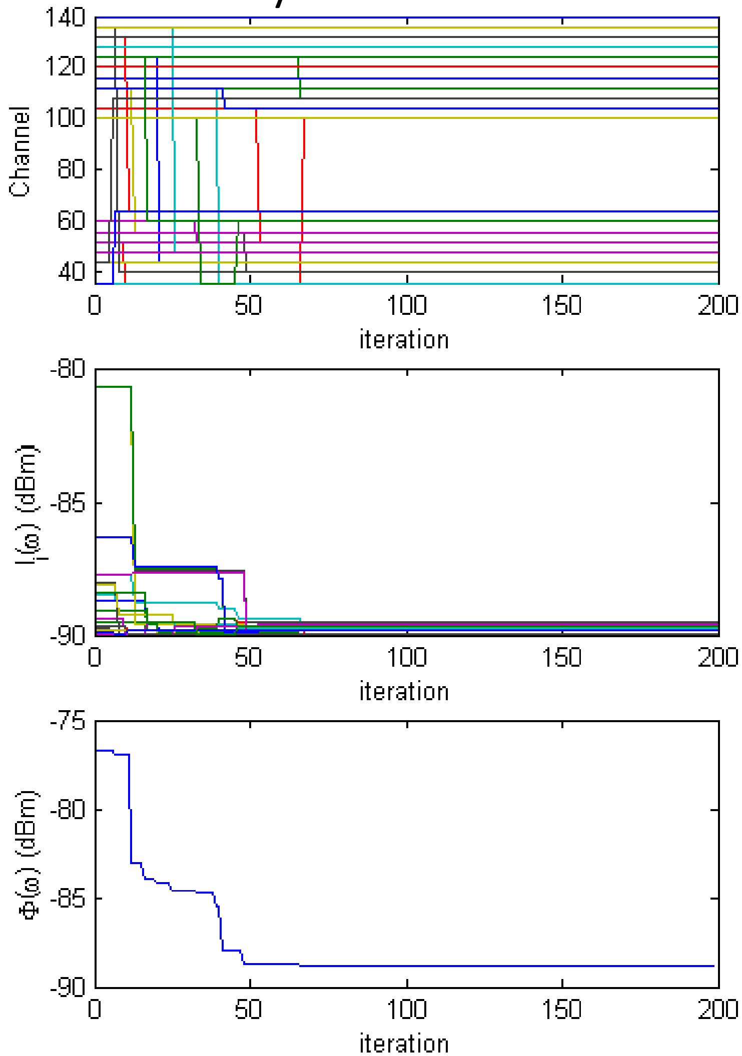


(highest frequency that improves)

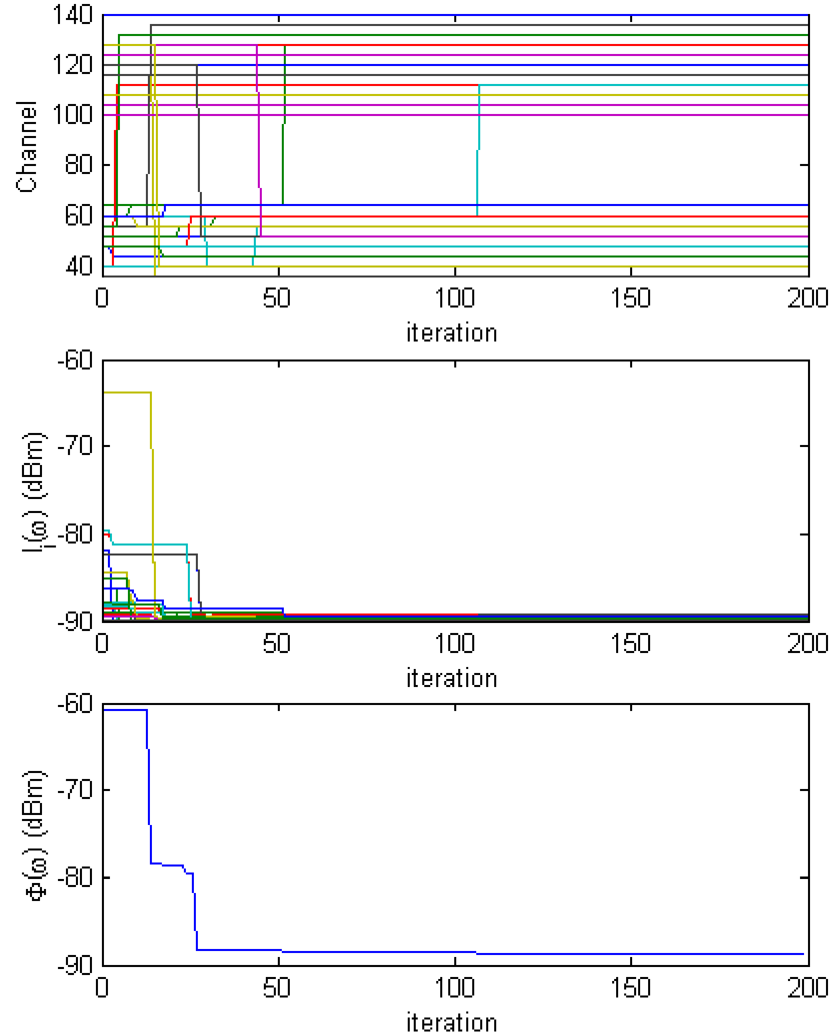


Policy Variations

By Channel

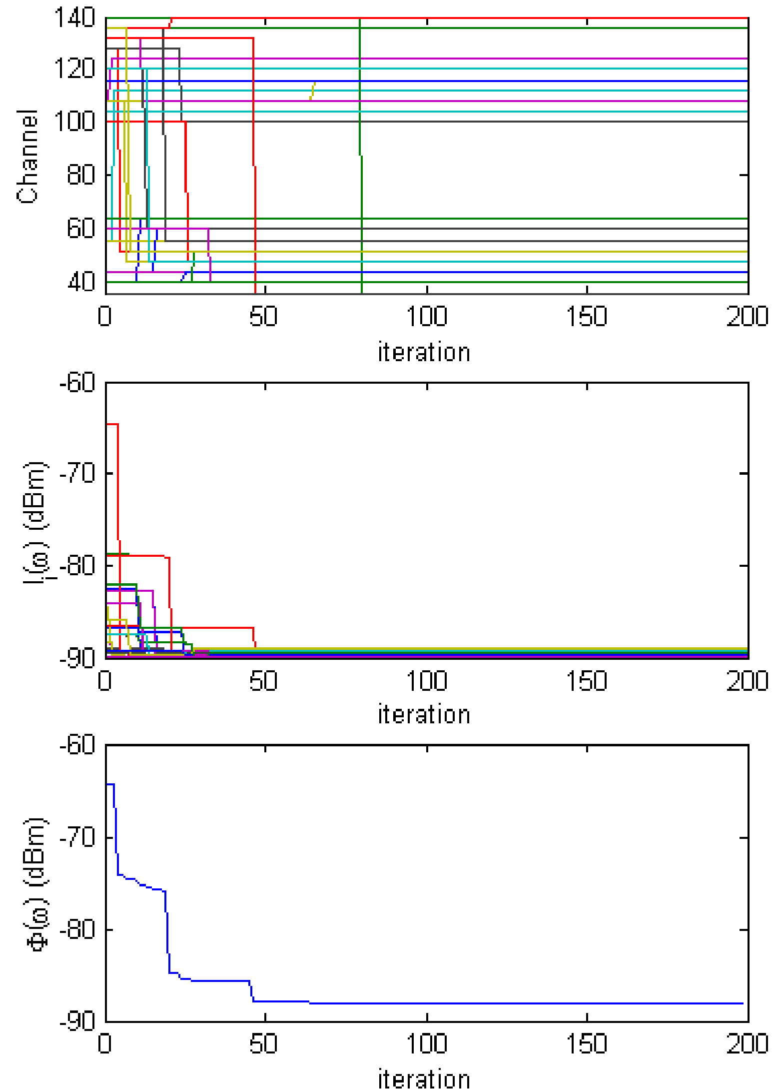


By Radio (10 radios only low channels)



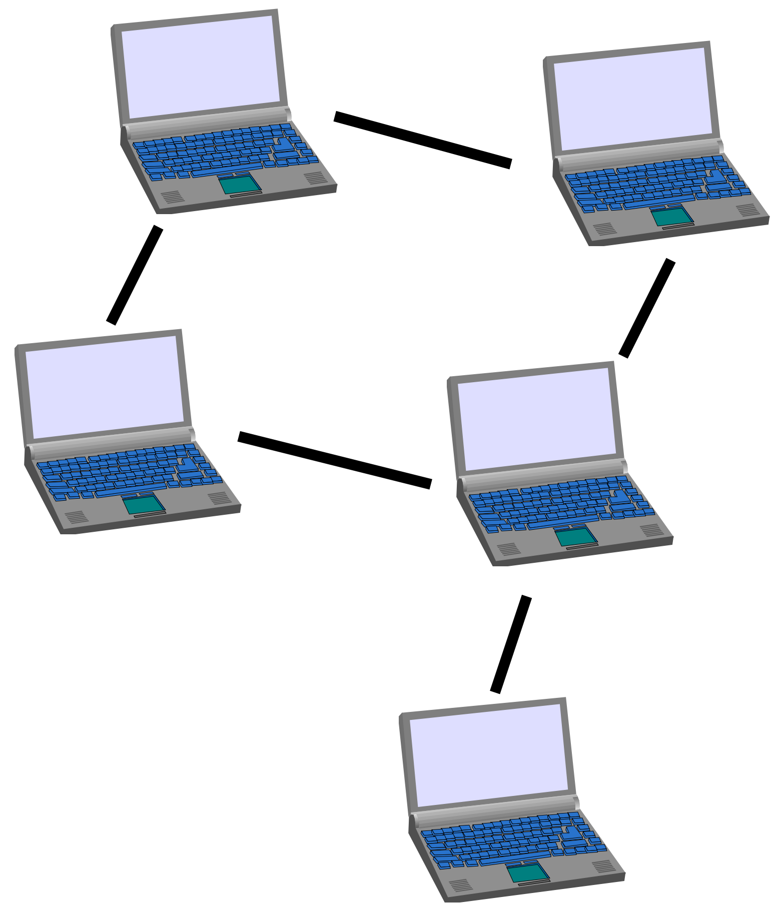
Local Frequency Preferences

- Each radio has a random real constant added to its observation of each channel
- Exact potential game
  - BSI + Self-motivated
- Equivalent to having legacy devices present
  - If legacy devices are transmitting at the same power as cognitive radios, then sum of interference of both cognitive radios and legacy radios is a monotonically decreasing function



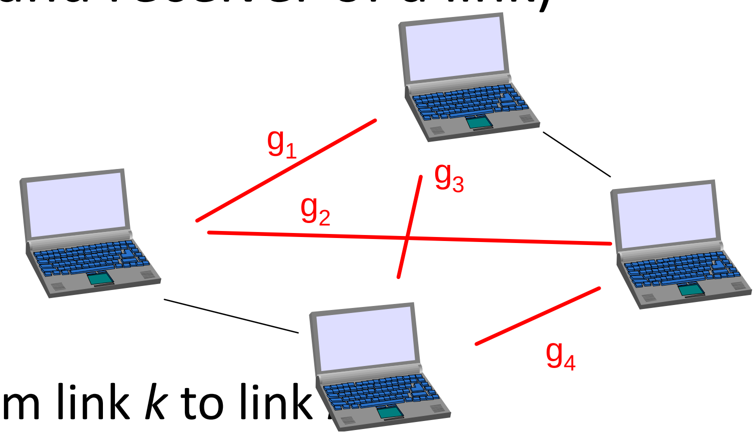
Problems Applying to Ad-hoc/P2P Network

- No clear master node
- No clear reason to privilege one observation over others
- Link gain asymmetry violates BSI (previous trick required all observations to be made at transmitters)
- Could designate master devices (ala Bluetooth) and then run the same algorithm as the infrastructure algorithm



Game Solution

- Define players as links
  - Both sides of a link collaborate to make a decision
  - Permits incorporation of observations from both radios
- Assume RTS/CTS signaling as before (or at least some signal which identifies sender and receiver of a link)
- Observation
  - Max interference from a link
  - Sum interference from a link



- Utility
  - P_{ki} = set of propagation paths from link *k* to link *i*

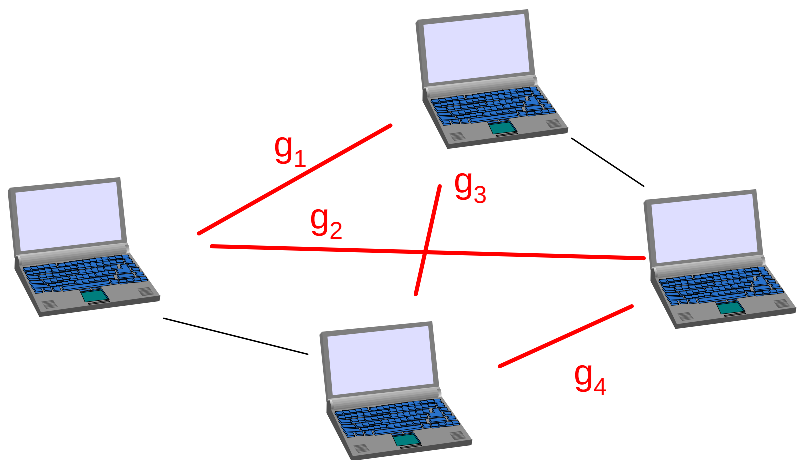
$$u_i(f) = -\sum_{k \in N \setminus i} \max_{j \in P_{ki}} \left\{ g_j p_k \sigma(f_i, f_k) \right\}$$

BSI for New Observations

- Utility

$$u_i(f) = -\sum_{k \in N \setminus i} \max_{j \in P_{ki}} \{ g_j p_k \sigma(f_i, f_k) \}$$

- Note p_k, σ are the same
- Max (g_{ik}) = Max (g_{ki})
- Sum (g_{ik}) = Sum(g_{ki})
- Thus we have BSI again



Comments on Options for P2P for BSI

- Three different combination techniques work
- Max, min very dependent on resolving addresses for interference table
- Sum doesn't need the addresses
- In fact

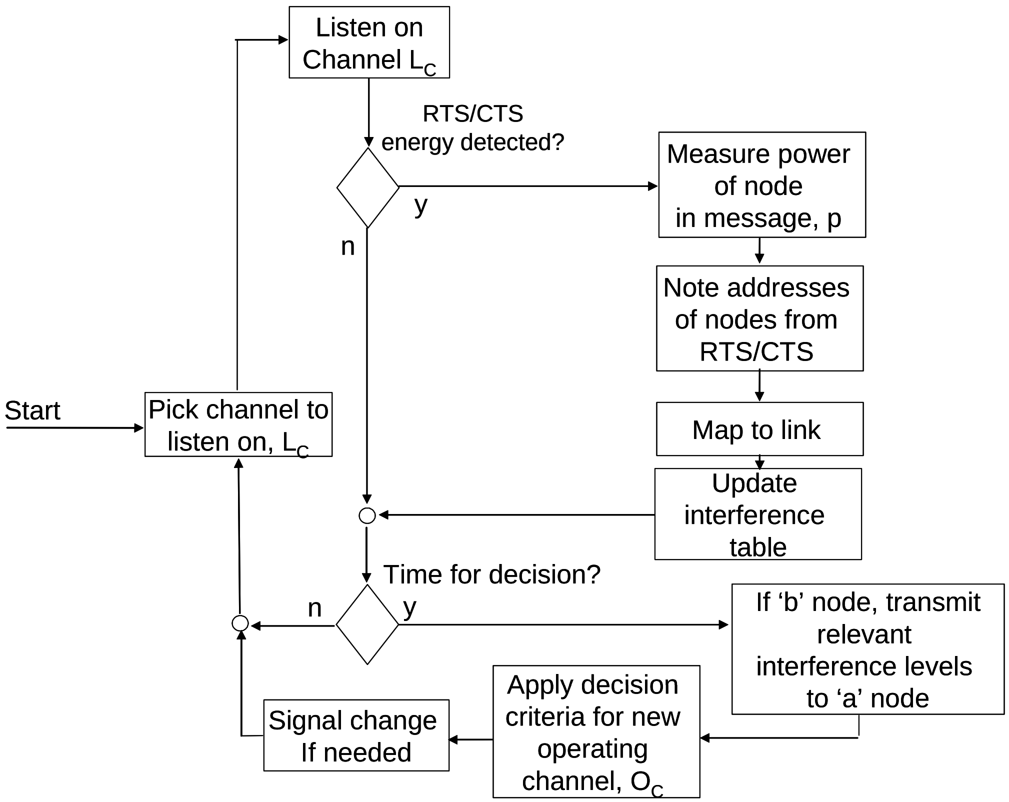
$$u_i(f) = -\sum_{k \in N \backslash i} \sum_{j \in P_{ki}} g_j p_k \sigma(f_i, f_k)$$ P_{ki} = set of propagation paths from link *k* to link *i*

- Is equal to

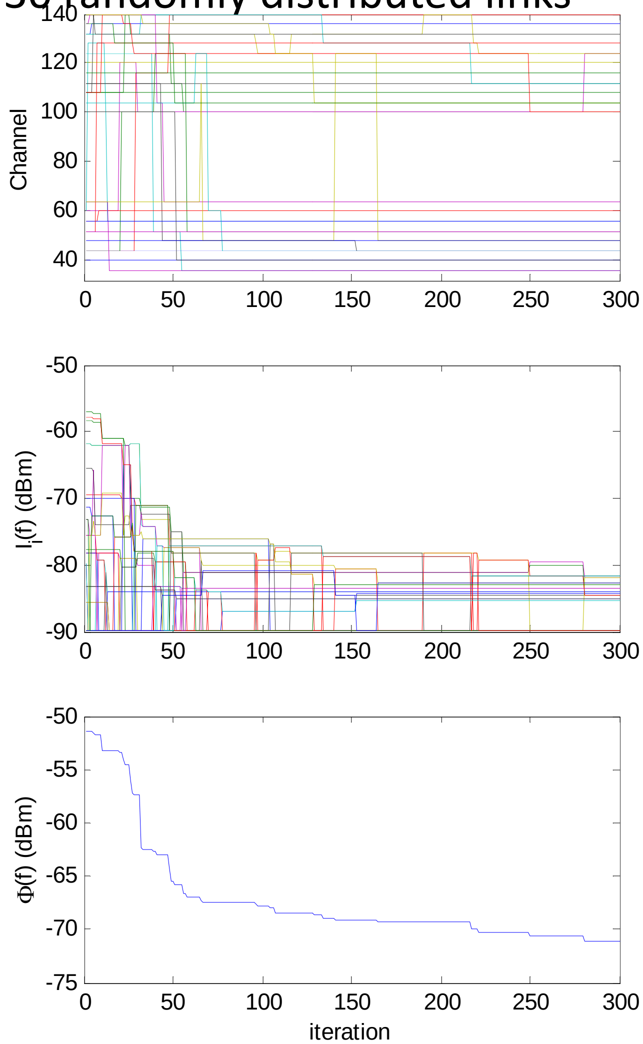
$$u_i(f) = -\left[\sum_{j \in N \backslash i} g_{ji_1} p_j \sigma(f_i, f_k) + \sum_{j \in N \backslash i} g_{ji_2} p_j \sigma(f_i, f_k)\right]$$

- Or the sum of aggregate interference observed between both sides of the link
- So can just take interference snapshots and sum together
- Addresses still help though
  - May not be able to listen to all channels simultaneously
  - Keep a more accurate representation of interference by removing interference contributions of adapted links from old channels once operation in a new channel is detected

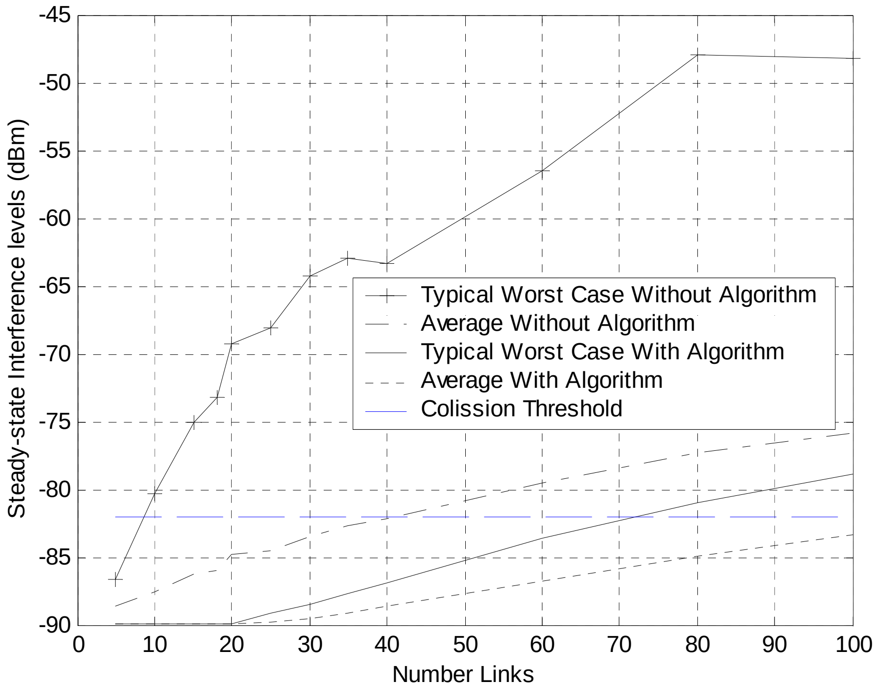
Peer-to-Peer DFS



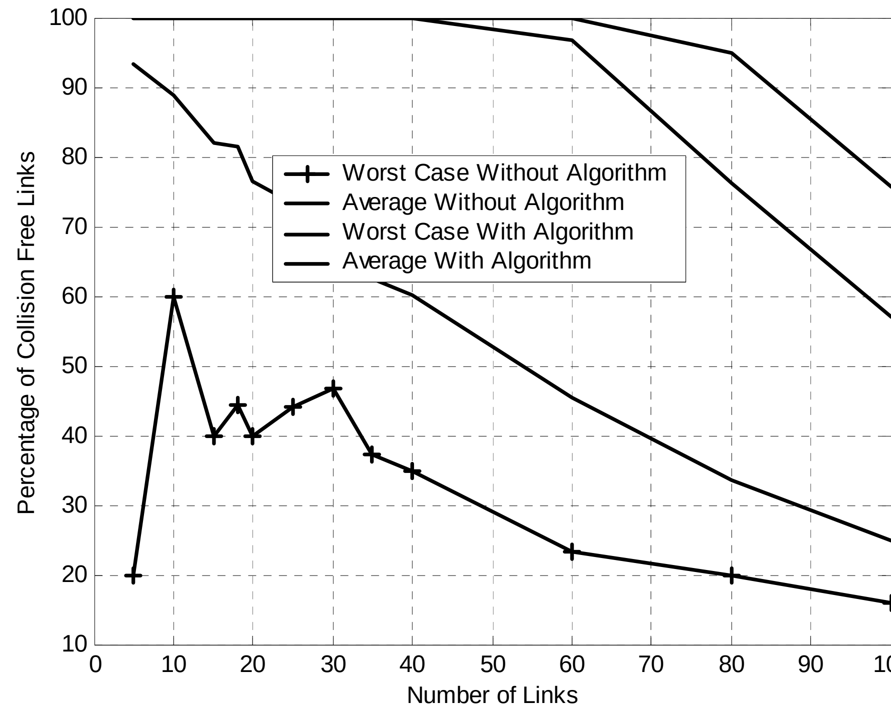
30 randomly distributed links



Aggregate Statistics



Nearly 30 dB reduction in worst case interference over random initial frequencies (average of worst link over each run)
10 dB reduction in average link interference
0.25 km x 0.25 km
Generally below -82 dBm collision threshold



By moving interference levels below the collision threshold, many more links can operate colission free
With random frequency 95% of all links are collision free for 5 links (average)
With algorithm, 95% level corresponds to 80 links

Comments on Implementation

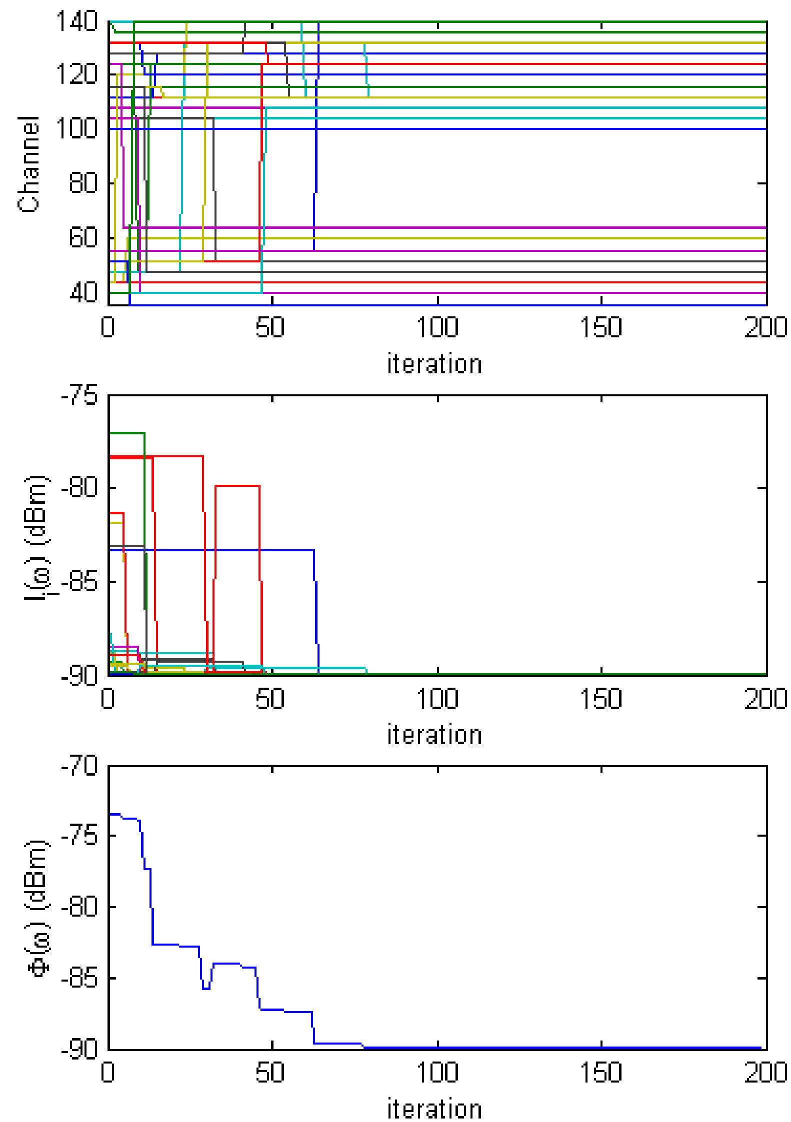
- As part of setting up the link, one of the two devices will have to be designated the master (‘a’) node.
- Not necessary to transmit interference measurements of all channels to ‘a’ node
  - Ranking of best candidates
  - a node can request subset
  - May be suboptimal at times, but suboptimal doesn’t damage convergence

Transmit Power Control and DFS

- Random distribution of powers (constant)
- Violates BSI as

$$g_{jk} p_j \rho\left(\omega_j, \omega_k\right) \neq g_{kj} p_k \rho\left(\omega_k, \omega_j\right)$$

- No longer monotonic
  – Forms a weighted potential game, specifically, weighted by the transmit powers (more later)



Problem with Varying Power Levels

- Different transmit power levels destroy BSI

$$g_{jk} p_j \rho(\omega_j, \omega_k) = g_{kj} p_k \rho(\omega_k, \omega_j) \quad \forall \omega_j \in \Omega_j, \forall \omega_k \in \Omega_k$$

- Still a weighted potential (for infrastructure – a mess for others), but weighted potential games don't form a linear space (bad for combining with other algorithms)
- In theory, could ask for transmit power levels and work it back, but don't want to incur the overhead (gets the $O(N^2)$ problem.
- How to do it as a zero-overhead solution?

Solution: Weight measured interference by your transmit power

- Infrastructure

$$u_i(f) = -p_i I_i(f) = -p_i \sum_{k \in N \setminus i} g_{ki} p_k \sigma(f_i, f_k)$$

- P2P or Ad-hoc (difference is bookkeeping)

$$u_c(f) = -\left[\sum_{i_m \in c} p_{i_m} \sum_{j \in N \setminus c} g_{ji_m} p_j \sigma(f_c, f_j) \right]$$

- Why it works:

$$g_{ki} p_k \sigma(f_i, f_k) \neq g_{ik} p_i \sigma(f_i, f_k)$$

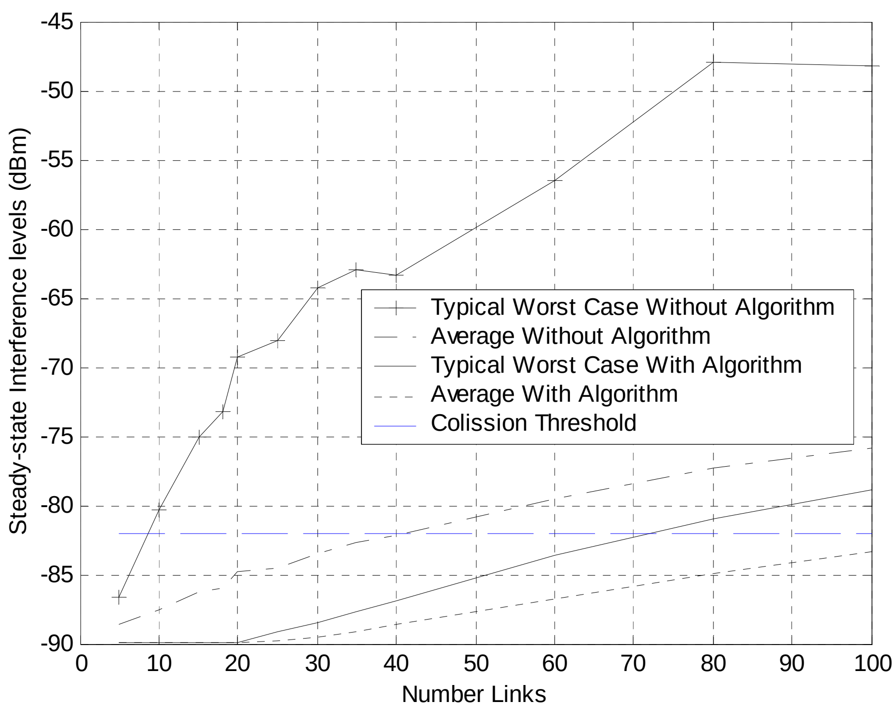
  – But

$$p_i g_{ki} p_k \sigma(f_i, f_k) = p_k g_{ik} p_i \sigma(f_i, f_k)$$

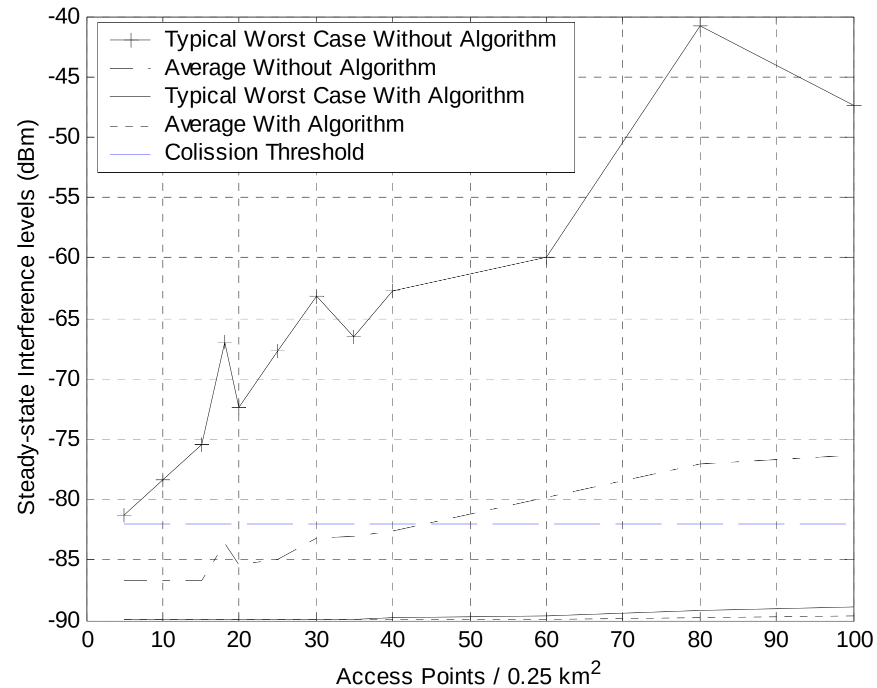
- We'll use this trick of multiplying asymmetric variables to create a symmetric variable a lot…

Adding Power Control to Ad-hoc Simulation

Transmit at max Power



Received Signal Power 16 dB > noise



5 trials

Activity Level Based DFS

Issue

- Not all interference is created equally
- Link may be near by on the same frequency but have very little traffic – could squeeze in there rather than some other channel with less signal power but greater activity
- Activity level can be estimated by rate at which a signal is detected, or total observed transmit duration
- Activity level is an asymmetric variable

Routing

Load-sensitive Routing Difficulties (Why Game Theory)

Bertsekas (1982)

> "The main idea in this scheme is to compute in real time an estimate of the minimum average delay per message for each origin-destination pair and to route messages along the current minimum estimated delay path. When this scheme was first implemented, it was noticed that it was prone to severe oscillations. This behavior is due to the fact **that delay estimates used to choose routes are themselves affected by the route choice with a feedback effect resulting**."

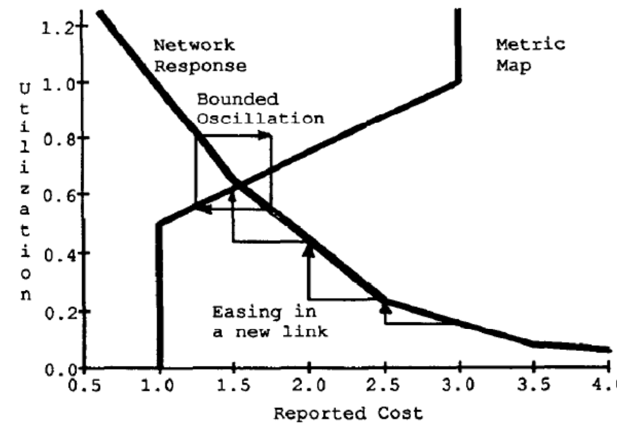
Khanna (1982)

> "The complex nature of the interactions between [Shortest Path First], the topology and the traffic matrix makes it difficult to analyze the system as a whole. In particular, note that the process of calculating equilibrium for a given link consists essentially of successively varying its cost and then recomputing the routes over it until its utilization converges to some value. Since a change in one link's routes affects the utilization and thus potentially the cost of other links, thus affecting its own choice of routes during the next iteration, any exact determination of equilibrium would have to consider this interplay between the links. Furthermore, this would have to be done simultaneously for all links, clearly a task of considerable complexity."

Load-sensitive Routing Difficulties (Why Game Theory)

Shaikh (1999)

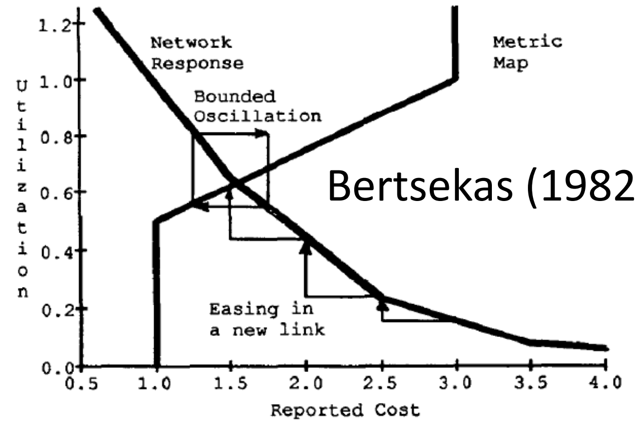
"By selecting paths that circumvent congested links, dynamic routing can balance network load and improve application performance. Despite these potential benefits, however, most backbone networks still employ static routing (e.g., based on routing protocols such as OSPF and IS-IS) because techniques for load-sensitive routing often lead to route flapping[3] and excessive control traffic overheads."



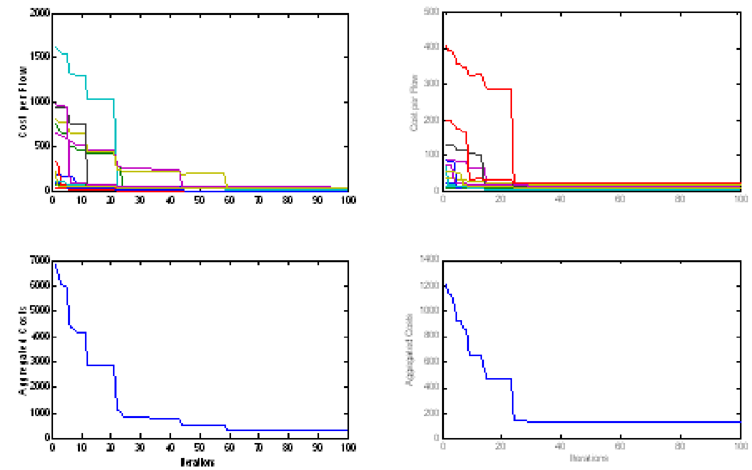
Bertsekas (1982)

Load-Sensitive Routing Results summary

- Traditional stability issues when load-sensitive
  - Interactions intractable for ARPANET
  - Turns out virtually every routing algorithm is load insensitive now
    - Hop count, link BW
- Generalized congestion game
  - Stable and load-sensitive
    - Ignores information distribution
  - Each edge is EPG
    - Action is contribution of traffic
  - Cost = 0 if not using edge
  - Path cost is some of edge costs



Bertsekas (1982)



Metric	Reference	Potential Game?
Hop count (Load-insensitive)		EPG
Traffic (uniform)		EPG
Traffic (nonuniform)		WPG
Delay (uniform traffic)	[Khanna_89]	EPG
Delay (nonuniform traffic)	[Khanna_89]	No
ETX (noise-limited)	[Couto_03]	EPG
ETX (interference-limited)	[Couto_03]	No[1]
ETT (noise-limited)	[Draves_04]	EPG[2]
ETT (interference-limited)	[Draves_04]	No[1]
WCETT (noise-limited)	[Draves_04]	No[3]
WCETT (interference-limited)	[Draves_04]	No

1. Is a potential game with uniform packet sizes and uniform traffic levels, but this seems unlikely in practice.
2. Can be a WPG for varying lengths.
3. Is a potential game for α =0 when ETT is also a potential game

Cognitive Radio Technologies
www.crtwireless.com

Basic Routing Modeling Assumptions

- Routing algorithms consist of three classes of processes:
  - Information gathering processes
    - How does a device know what it needs to know across the network?
    - Ex: Flooding, table exchange
  - Decision processes
    - Generally choose path that minimizes sum of edge (link) costs over path
    - $u_p(r) = \sum_{e \in r_p} c_e(r)$
  - Decision execution processes
    - How is decision executed?
- Each end-to-end flow of data has its own "path manager" that attempts to find minimum cost sequence of edges from one end to the other
  - Doesn't exist physically
  - More of an emergent phenomenon
- Other
  - Fixed network topology
  - Perfect knowledge
  - Perfect implementation

Focus on interactions

Our basic routing game model

- *P* – Set of "path managers"
- r_p – route chosen by path manager *p*.
  - Collection of edges
- R_p - the set of all possible feasible routes
  - Implies no loops, broken paths in route set
- Routing action space

$$R = R_1 \times R_2 \times \cdots \times R_{|P|}$$

  - Implies no condition where if path *k* uses edge *e*, then path *m* cannot use edge *e*

- Route vector
  - Choice of routes by each path manager

$$r = \left(r_1, r_2, \ldots, r_{|P|}\right)$$

- Edge cost
  - Possibly function of routes not using *e*

$$c_e : R \rightarrow \mathbb{R}$$

- Path manager objective

$$u_p\left(r\right) = -\sum_{e \in r_p} c_e\left(r\right)$$

  - Minimize sum of costs on all edges used in own path

Congestion model EPG for routing

- Congestion game
- Cost model
  - $u_i(a) = -\sum_{f \in a_i} c_f(\sigma_f(a))$
  - $\sigma_f(a) = \#\{i \in N : f \in a_i\}$
- EPF

$$V(a) = -\sum_{f \in \bigcup_{i=1}^{n} a_i} \left(\sum_{k=1}^{\sigma_e(a)} c_f(k) \right)$$

- Physical meaning
  - Steady-states (choice of facilities) are local minimizers for aggregation of facility costs

- Routing game

$$u_p(r) = -\sum_{e \in r_p} c_e(\sigma_e(r))$$

$$\sigma_e(r) = \#\{p \in P : e \in r_p\}$$

- EPF

$$V(r) = -\sum_{e \in \bigcup_{p=1}^{n} r_p} \left(\sum_{k=1}^{\sigma_e(r)} c_e(k) \right)$$

- Physical meaning
  - Steady states (choice of edges for each path) are local minimizers of edge costs.

Looking at the Congestion Model Sideways

- Could be equivalently written as sum over all edges, but with condition that p receives payoff (cost) of 0 for not using an edge

$$u_p(r) = -\sum_{e \in r_p} c_e(\sigma_e(r)) \Leftrightarrow -\sum_{e \in E} c_{e,p}(\sigma_e(r))$$

  - Possibly different edge costs for different paths
  - Implicit requirement that $c_{e,p}(\sigma_e(r)) = 0, e \notin r_p$

- Potential function could be written as

$$V(r) = -\sum_{e \in \bigcup_{p=1}^{n} r_p} \left(\sum_{k=1}^{\sigma_e(r)} c_{e,p}(k) \right) \Leftrightarrow \sum_{e \in E} V_e(r)$$

  - where

$$V_e(r) = \left(\sum_{k=1}^{\sigma_e(r)} c_{e,p}(k) \right)$$

  - Requirement that

$$V_e(r) = 0, e \notin r$$

More generalized congestion model

- Define “edge game”

$$\Gamma_e = \left\langle P, R_e, \left\{ c_{e,p}\left(r_e\right) \right\} \right\rangle$$

  – just based on choices related to edge *e* (*r* is then a binary vector of size |*P*|

- Restrict $c_{e,p}$ such that edge game is an EPG and

$$c_{e,p}\left(\sigma_e\left(r\right)\right) = 0, e \notin r_p$$

- Then network wide game

$$\Gamma^N = \left\langle P, R, \left\{ \sum_{e \in E} c_{e,p}\left(r_e\right) \right\} \right\rangle$$

  – is an EPG via linearity property

Fun Insights

- Edge costs do no have to be solely a function of # of users
  - Cost could vary by path
  - More complicated cost functions
- Since just need a linear combination could weight "critical edges"
  - Would have to be done uniformly across the network
- Applicability of coordination games limited
  - Edge costs have to be of form

$$c_{e,p} = \begin{cases} 0 & r_e \neq \vec{\mathbf{1}} \\ K & r_e = \vec{\mathbf{1}} \end{cases}$$

- Applicability of dummy games dubious
  - $c_{e,p} = 0$
- BSI, MSI, Self-interested, and original congestion cost games would work
  - Could combine edge costs
    - Interactive cost (BSI / MSI) + self cost
    - Self-cost + self-cost
- Implies simple method for identifying if routing algorithm (as defined by sum of edge costs) is an EPG
  - Show edge cost game is
    - an EPG
    - Satisfies zero condition

Examples

- Self-interested edge cost

$$c_{e,p}(r) = \begin{cases} 0 & r_{e,p} = 0 \\ K_{p,e} & r_{e,p} = 1 \end{cases}$$

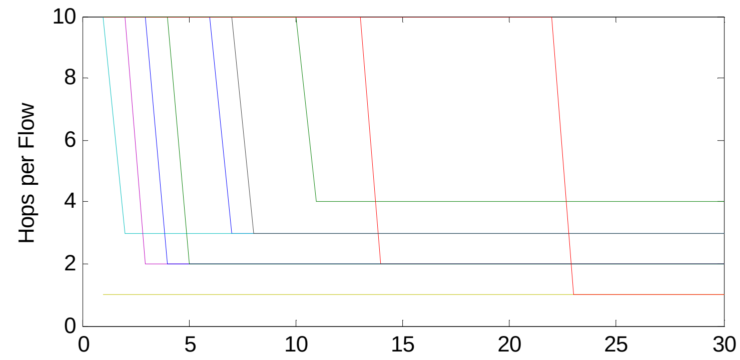
- Network-wide EPG

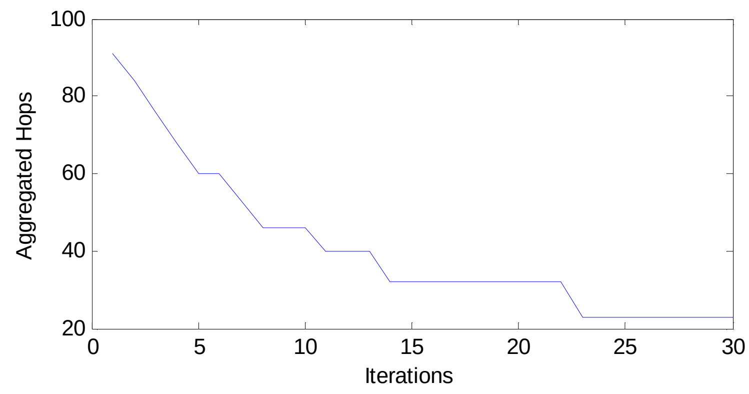
$$V(r) = -\sum_{e \in E} \sum_{p \in P} c_{e,p}(r)$$

- Traditional fixed path cost metric
  - Let $K_{p,e}$ be any fixed common constant (e.g., power or inverse of link throughput rate)
    - Route chocies
- Interference Aware routing
  - Let $K_{p,e}$ be a constant for all users of *e* and be the interference seen by *e*.
    - Interference measurement has to be independent of route choices
  - Then route choices will converge to routes that minimize aggregation of measured interference
- If $K_{p,e}$ is the same for all *p*
  - Really just a different way of proving Bellman Ford relaxation across network
  - Technically an example of a congestion game where cost is independent of #
- If not, then could have different edges privileging different classes of traffic
  - Mix of diffserv clouds
- Could've been a common constant for all + a path-specific offset
  - Power + traffic-specific model
- Lots of these implications are not necessarily new algorithms, but we've pulled them into the EPG framework

Metric: Hops

- Edge cost $c_{e,p}(r_e)=\begin{cases}1 & e\in r_p \\ 0 & e\notin r_p\end{cases}$
- Self-interested
  - Means *no* interaction
- Has edge exact potential
  - $V_e(r_e)=\sum_{i\in N:e\in r_i} 1$
- End-to-end potential
  - $V(r)=\sum_{e\in E}\sum_{i\in N:e\in r_i} 1$
  - Set of paths that minimize hops is an equilibrium





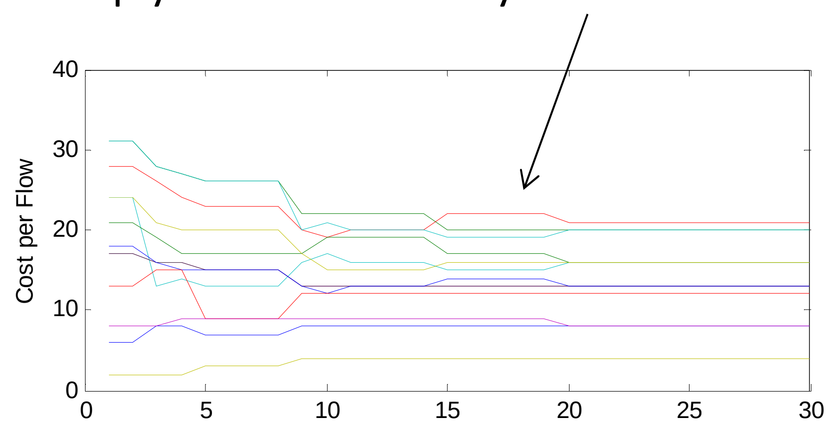
Edge Cost: Traffic (Uniform)

- Edge cost

$$c_{e,p}(r_e) = \begin{cases} \sum_{i \in N : e \in r_i} t_i & e \in r_p \\ 0 & e \notin r_p \end{cases}$$

$$u_p(r) = \sum_{e \in r_p} \sum_{i \in N : e \in r_i} t_i$$

Monotonic potential function does not imply monotonic utility functions

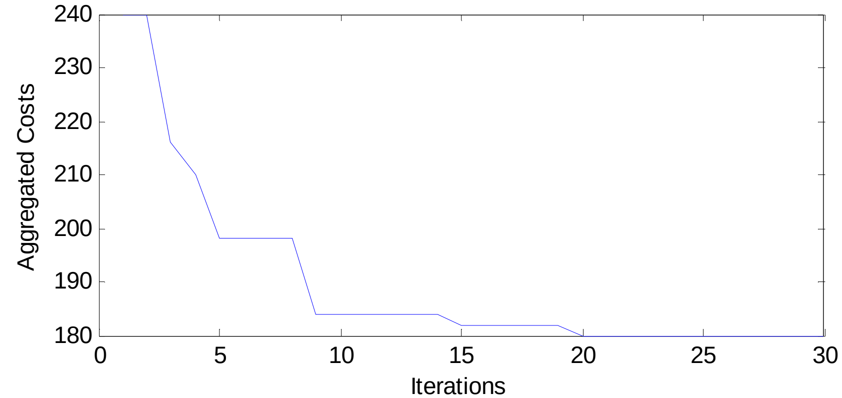


- If all t_i are the same, then it's a congestion game

$$u_i(a) = \sum_{f \in a_i} c_f\left(\sigma_f(a)\right)$$

$$\sigma_f(a) = \#\{i \in N : f \in a_i\}$$

$$c_{e,p}(r_e) = \sigma_e(r_e)\, t_i$$



$$V_e(r) = \sum_{p \in N : e \in r_p} t_p$$

Edge Cost: Traffic (Non-Uniform)

- No longer a congestion game

$$c_{e,p}\left(r_e\right)=\begin{cases}\sum_{i\in N:e\in r_i} t_i & e\in r_p \\ 0 & e\notin r_p\end{cases}$$

  - Depends on who, not just #
- Also not an EPG
  - Not differentiable @ 0
  - $\sum_{i\in N:e\in r_i} t_i$ is not an edge exact potential
- Ordinal transformation
  - $$c_{e,p}\left(r_e\right)=t_p\sum_{i\in N:e\in r_p} t_i$$
  - Weight by own traffic
  - Makes a BSI game
  - Edge exact potential

- End-to-end

$$u_p\left(r\right)=\sum_{e\in r_p} t_p \sum_{i\in N} t_{i,e}$$

$$V\left(r\right)=\sum_{i\in N:e\in r_i}\sum_{i\in N:e\in r_i} t_i t_j/2$$

- More importantly, an ordinal transformation of original end-to-end utility

$$u_p\left(r\right)=t_p\sum_{e\in r_p}\sum_{i\in N} t_{i,e}$$

- Original is a WPG with weights: $w_p = 1/t_p$

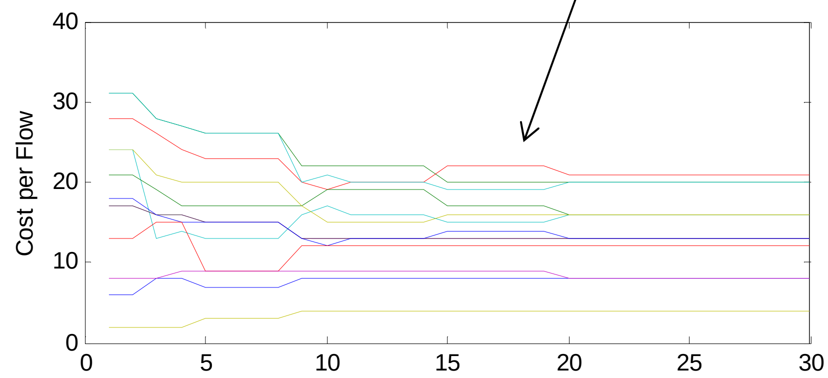
Edge Cost: Traffic (Non-Uniform)

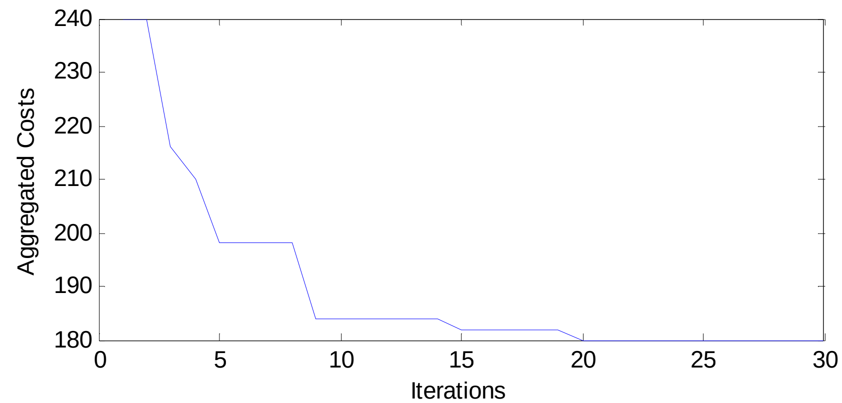
- No longer a congestion game

$$c_{e,p}\left(r_e\right) = \begin{cases} \sum\limits_{i \in N: e \in r_i} t_i & e \in r_p \\ 0 & e \notin r_p \end{cases}$$

- But can show as weighted a potential game via ordinal transformations (diss) and congestion game extension

Monotonic potential function does not imply monotonic utility functions



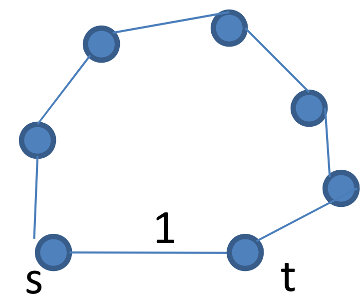


Edge Cost: Uniform Traffic + Hops

- Clear pathologies to just traffic
  - Route through *a lot* of hops if no traffic
  - Prefer to also add in some weighting of hops

$$c_{e,p}\left(r_e\right)=\begin{cases}1+\sum_{i\in N:e\in r_i} t_i & e\in r_p\\ 0 & e\notin r_p\end{cases}$$

- Hop count (or any *fixed* edge cost) is a self-interested edge game.
- Uniform traffic yields congestion cost



- Use linear combination on edge potentials
  - $V_e\left(r\right)=V_{e,hops}+V_{e,cong}$

$$=\sum_{i\in N:e\in r_i} 1+\sum_{k=1}^{\sigma_e(r)} c_e\left(k\right)$$

  - Implies that we could weight hops against traffic

Edge Cost: Non-Uniform Traffic + Hops

- Now edge game is linear combination of WPG and EPG
  – Result may not be a potential game
- But same traffic weighting trick applies

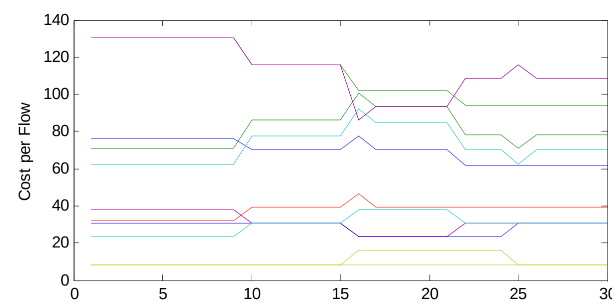
$$\tilde{c}_{e,p}\left(r_e\right) = t_p c_{e,p}\left(r_e\right) = t_p \begin{cases} 1 + \sum_{i \in N: e \in r_i} t_i & e \in r_p \\ 0 & e \notin r_p \end{cases}$$

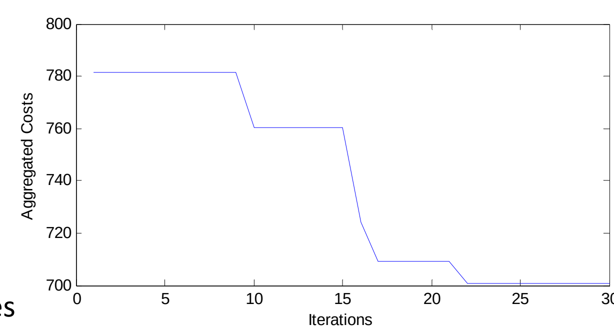
- Recognize as linear combination of self-interested and BSI

– $u_p\left(r\right) = t_p \sum_{e \in r_p} \left(1 + \sum_{i \in N: e \in r_i} t_{i,e}\right)$

$$V\left(r\right) = \sum_{e \in E} \sum_{i \in N: e \in r_i} \sum_{i \in N: e \in r_i} t_i t_j / 2 + \cdots$$

$$+ \sum_{e \in E} \sum_{i \in N: e \in r_i} t_i$$





Metric: End-to-End Delay

$$c_{e,p}(r_e) = \begin{cases} 1/\left(T_e - \sum_{i \in N: e \in r_i} t_i\right) & e \in r_p \\ 0 & e \notin r_p \end{cases}$$

- Not differentiable @ 0
- Can't apply second derivative test
- Scaling by t_i fails second derivative test
- Edge cost ordinally equivalent to

$$\sum_{i \in N} t_i - T_e$$

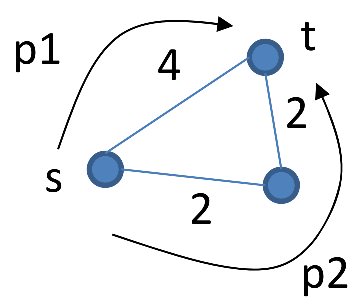
- However, is

$$u_p(r) = \sum_{e \in r_p} \sum_{i \in N} (t_i - T_e)$$

- not ordinally equivalent to

$$u_p(r) = \sum_{e \in r_p} 1/\left(T_e - \sum_{i \in N: e \in r_e} t_i\right)$$

- For example: Te = 5



Modified metric is indifferent
Original prefers p2 to p1
1/1 > 1/3 + 1/3

Expected Transmissions (EXT)

- Effectively Hop + Expected # of collisions
- Highly dependent on MAC
- If collision free:
  - Scheduled ala 802.16h
  - Expected Transmissions is a fixed number
  - Just like weighted hop counts
- If not collision-free, gets messy fast
  - Sum of (p_not other collisions) * (p_this collision)
  - Not exactly BSI or MSI due to prob of no collision
- Sum of probability of pairwise collisions is an EPG if up/down link shares band
  - Similar in thrust
  - Different equilibria

Technique 2: Cost Products with metric redefinition (ETT approx.)

- Suppose modified metrics are defined such that

$$w_p(r,s) = w_m(r,s) = u_m(s)u_p(r)$$

- Then if both routing and spectrum management games are MSI games, then result is also an MSI game
  - E.g., abcd = bcda = cdba…

$$t_e t_f = \sum_{p\in P}\left(t_p r_{e,p}\right)\times\left(t_p r_{f,p}\right)$$

- Example

$$c_{e,p}(s,r) = I_e^t(s) =$$
$$t_e \sum_{f\in E\setminus E_m} t_f \sum_{n_1\in e}\sum_{n_2\in f} p_f g(n_1,n_2)\sigma(s_e,s_d)$$

where original routing metric was effectively sum of *tp x tr* terms over in-band edges over all path managers

Expected Transmission Time (ETT)

- Expected time to complete a transmission across a link.
- Effectively (varies by MAC)
  - ETX X Packet length / link bandwidth
- If packet (frame) length is fixed / common, then ETT is an EPG if ETX is an EPG
- If packet length varies with flow, then ETT is a WPG if ETX is an EPG

- Also a Weighted Transmission Time

$$u_p(r) = (1-\alpha)\sum_{e\in r_p} c_{e,ETT}(r) + \alpha W_j$$

  - Effectively ETT weighted with worst delayed link
  - Since worst delayed link varies between paths, should almost always violate symmetries needed for potential game

Summary of Routing Mapping

- Several common metrics are EPG
  - Hop, traffic
- Some of the more popular metrics for wireless ad-hoc are not EPG
  - There are similar (but not ordinally equivalent) substitutes that are EPG
    - Delay, ETX, WCETT

Cross-Layer Approaches

General Problem

- Assume two distributed algorithms

$$\Gamma^1 = \left\langle N, A, \{u_i\} \right\rangle \quad \Gamma^2 = \left\langle M, B, \{v_i\} \right\rangle$$

- Possible dependencies of choices
  - Performance depends on choices of other algorithm
    - SINR targeting power control and interference avoidance frequency selection
  - Actions depend on choices of other algorithm
    - DSA and frequency selection
    - Power control (topology) and routing
  - Players depend on choices of other algorithm
    - Edge congestion games and frequency selection

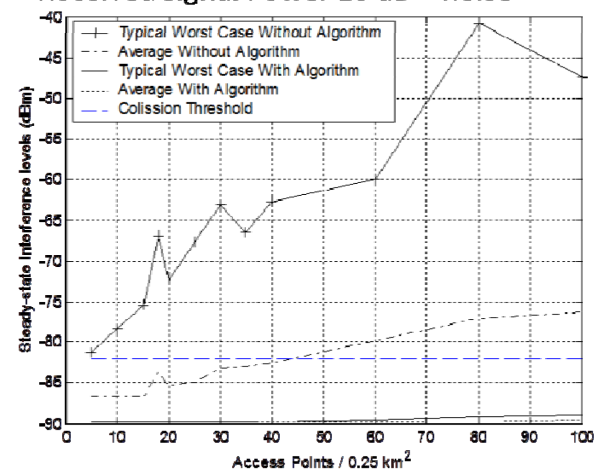
Broad Coupling Categories

- Observations from multiple layers, single parameter
- Perfectly Decoupled
  - no interaction, each process converges on own
- Unilaterally Decoupled
  - One algorithm influences the other, but not vice versa
- Closely Coupled
  - Each algorithm influences the other

Unilaterally Decoupled

- Concept – one algorithm depends on the other, but not vice-versa
- Previous examples within this tutorial
  - Power control to close link, frequency selection for interference mitigation
  - Beamforming for target SNR, frequency selection for

Received Signal Power 16 dB > noise

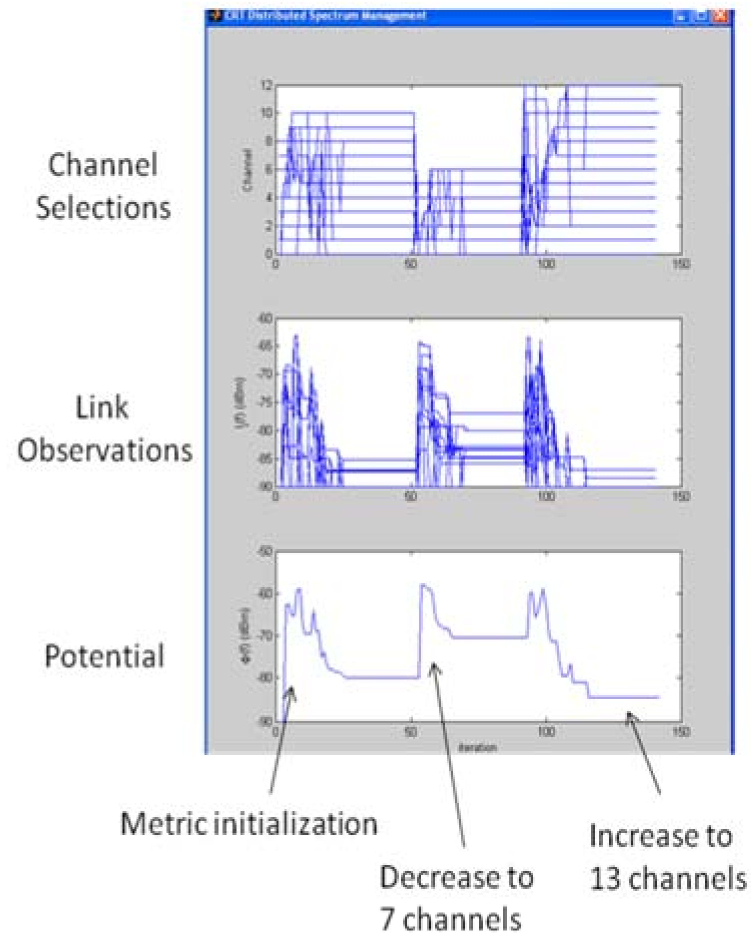


5 trials

$$u_i(f) = -I_i(f) = -\sum_{f_i \in SC_i} p_{f_i} \sum_{k \in N \setminus i} g_{ki} p_{k,f_k} \sigma(f_i, f_k)$$

DSA + Spectrum Management

- Model:
  - W/O DSA $\Gamma = \langle N, A, \{u_i\} \rangle$
  - W/ DSA $\Gamma = \langle N, A^t, \{u_i\} \rangle$
  - Primary users impact available adaptations (channels)
- PU effectively parameterizes game, but structure remains
  - Converges while A^t is constant
- Different if primary users react to secondary users
- Similar for DSA + load-sensitive routing



Easiest Solution (when it works)

- Define new game as

$$\Gamma^1 = \left\langle N, A, \{u_i\} \right\rangle \quad \Gamma^2 = \left\langle M, B, \{v_i\} \right\rangle$$

$$\Gamma^3 = \left\langle M \cup N, A \times B, \{w_i\} \right\rangle$$

$$w_i = u_i(a) + v_i(b)$$

- Assume two games are EPG, then:
  - If perfectly decoupled, then remains EPG
  - If unilaterally decoupled, then weighting first by more than the largest possible utility change is an OPG
- Messy with action / player dependencies or if closely coupled (in reality, *v* depends on *a* and *u* depends on *b*)
  - Focus of next set of slides

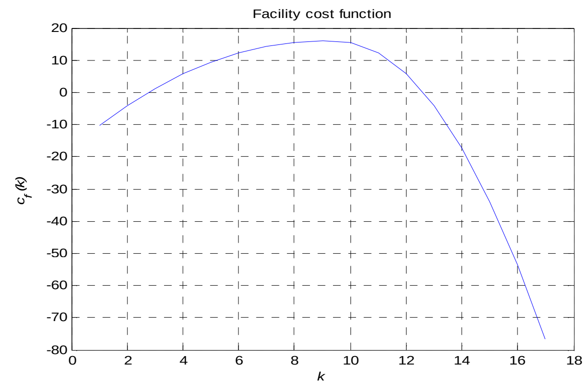
Example with Metrics from Multiple Layers

Multi-metric Potential Game Example

- Density (#one-hop and two-hop neighbors)

$$u_i(d) = \sum_{f \in d_i} c_f\left(\sigma_f(d)\right) \qquad V(d) = \sum_{f \in \bigcup_{i=1}^{n} d_i} \left(\sum_{k=1}^{\sigma_f(d)} c_f(k) \right)$$

Example cost function



Number nodes on channel @ node

- Load (traffic) Sensitive

$$u_{i,traffic}(d) = -traff_i \sum_{i \in N_i} traff_j \sigma(d_i, d_j) \qquad \sigma(d_i, d_j) = \begin{cases} 1 & d_i = d_j \\ 0 & d_i \neq d_j \end{cases}$$

$$V_{traffic}(d) = \sum_{i \in N} traff_i \sum_{j \in N_i} traff_j \sigma(d_i, d_j) / 2$$

- Interference

$$u_{i,\text{int}}(d) = f_i(d_i) \qquad V_{\text{int}}(d) = \sum_{i \in N} f_i(d_i)$$

- Combined

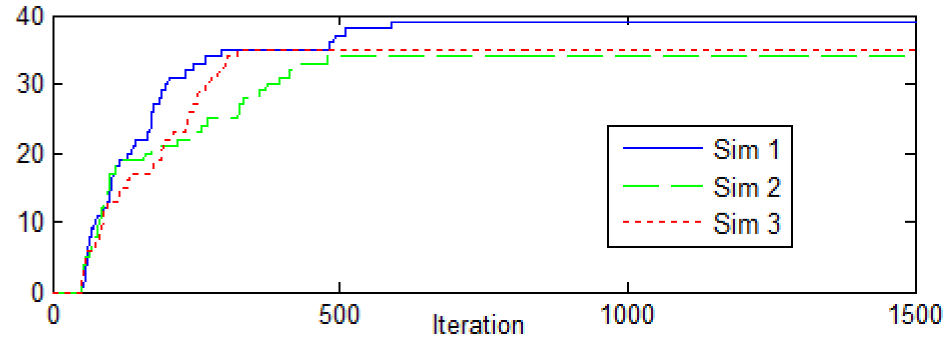
$$u_{i,combined}(d) = \alpha u_{i,topo}(d) + \beta u_{i,\text{int}}(d) + \gamma u_{i,traffic}(d)$$

$$V_{combined}(d) = \alpha V_{topo}(d) + \beta V_{\text{int}}(d) + \gamma V_{traffic}(d)$$

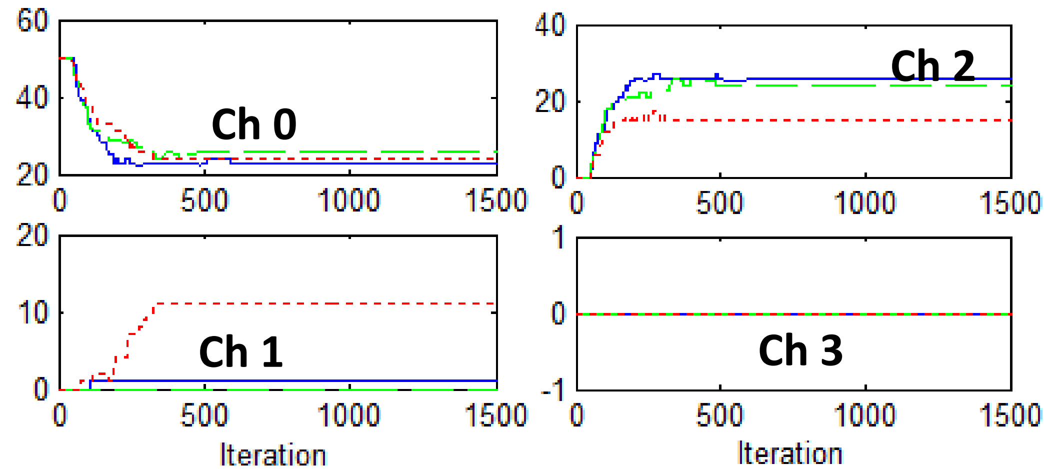
Implications of Stability

- Sensitive to local conditions, without runaway adaptations
  - Means fewer adaptations in MANET setting
- Importance of persistent conditions for costly adaptations
  - Ignore traffic spikes
  - Not intermittent interference

Cumulative # Adaptations

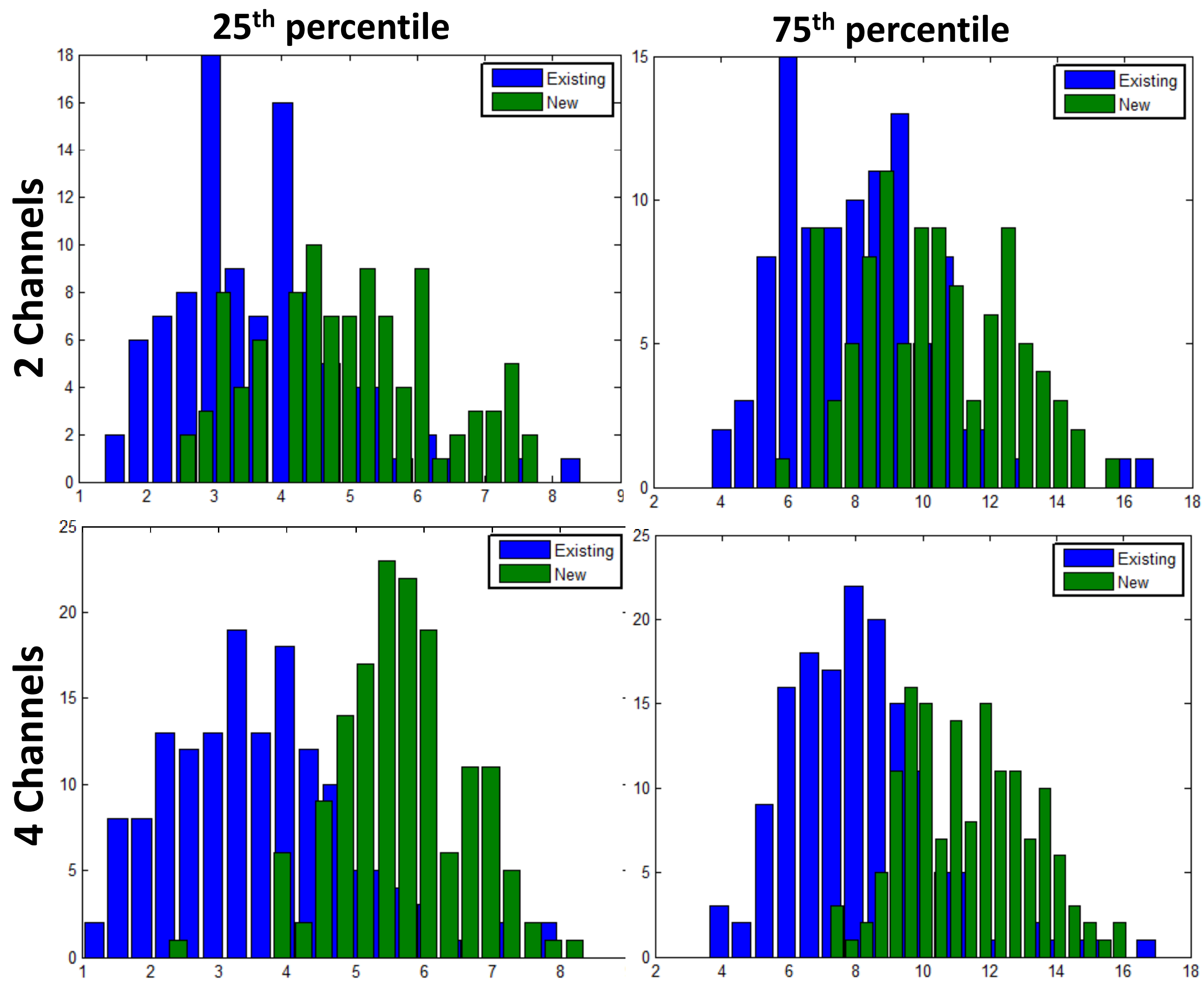


Node Channel Distribution



Awareness of Local Conditions Improves Performance

- Link margin improvement
  - –**Median: 2-5 dB**
  - –**Worst node: 5-16 dB**
- Throughput Increase
  - –**Median: 30-54%**
  - –**Worst node: 50-145%**
- Sim Params (on right)
  - – 50 nodes
  - – Varying number of interferers
  - – High traffic demand



Messier Coupling

Dynamic / load-sensitive routing is always messier

Induce a network-wide function

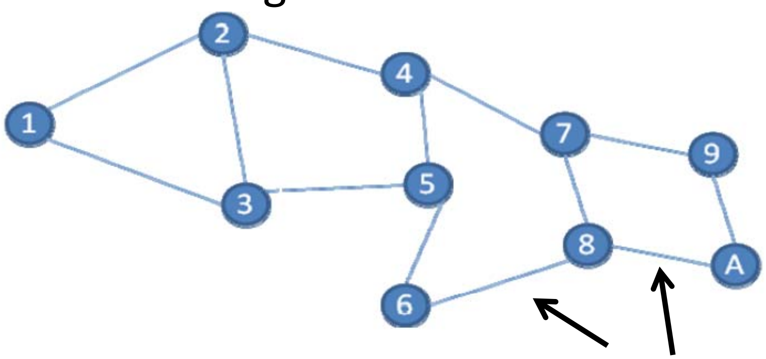
* **Theorem 1**: Let $\Gamma^1 = \langle N, A, \{u_i\} \rangle$ and $\Gamma^2 = \langle M, B, \{v_j\} \rangle$ be two different exact potential games with potentials V^1 and V^2, respectively. Then $\Gamma^3 = \langle M \cup N, A \times B, \{V^2(b)u_i(a), V^1(a)v_j(b)\} \rangle$ is an exact potential game with potential $V^3 : A \times B \to R = V^1 V^2$.

- Proof based on showing yet another MSI game
- Downside – requires system-wide knowledge to calculate potential functions
- Further research angle – what happens with limited visibility for utility functions, e.g., with hazy sighted routing?

Generalized Congestion Game (Repeat)

- Concept:
  - Interaction on each edge is its own EPG
  - Route cost is sum over all edges in route
  - Payoff is **zero** for an edge if not routing over it

$$u_{N,p}(r) = -\sum_{e \in r_p} c_{e,p}(r_e)$$



Each edge is its own game

Given edge e, define the game played on e as $\Gamma_e = \langle P, R_e, \{c_{e,p}(r_e)\}\rangle$ where R_e is R restricted to the each path manager's choice to use e or not so that $c_{e,p}(r_e)$ is only a function of those paths that use r_e and only the parts of those paths that could go over e. Further, assume that Γ_e satisfies the following pair of properties.

(P1) Edge EPG Γ_e is an exact potential game, i.e., there exists a $V_e : R_e \to \mathbb{R}$ such that $\forall p, q \in P, \forall r, s \in R$

$$c_{e,p}(r_{p,e}, r_{-p,e}) - c_{e,p}(s_{p,e}, r_{-p,e}) = V_e(r_{p,e}, r_{-p,e}) - V_e(s_{p,e}, r_{-p,e}) \quad (29)$$

where $r_{-p,e}$ refers to the choice of routes by all path managers in $P \backslash p$ from R_e.

(P2) Zero If $e \notin r_p$, then $c_{e,p}(r_{p,e}, r_{-p,e}) = 0$.

In other words, if p does not route through e, the cost to p of "using" must be 0. This has the implication that dummy games where $c_{e,p}(r_p, r_{-p})$ are functions of r_{-p} but not r_p will not simultaneously satisfy both P1 and P2 unless $c_{e,p}(r_p, r_{-p})=0$ for all r_{-p} (else changing r_p can change the value of $c_{e,p}(r_p, r_{-p})$ which makes it no longer a dummy game).

- Applications:
  - Any prior load-insensitive
  - Any prior congestion game
  - Sum of aggregate traffic
    - Also various multilinear, polynomial and self-interested variations
  - Where the edge cost is an MSI game
    - Useful with spectrum allocation (in a moment)

Inducing MSI in Edge Games

- First simplify player combination by assuming all non-routing players are notionally controlling an edge
- Spectrum management and routing utilities

$$u_p\left(r_p, r_{-p}, s_e, s_{-e}\right) = -\sum_{e\in E}\sum_{p\in P}\sum_{q\in P}\sum_{f\in E\setminus e}\left[t_{p_e} t_{q_f} I_{ef}\left(s_e, s_f\right)\right]$$

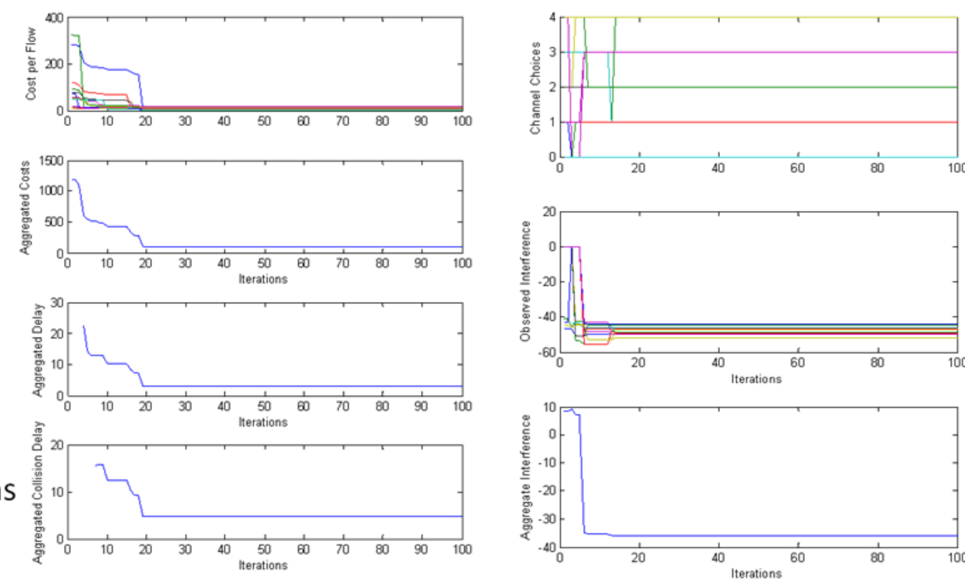
$$u_m\left(r, s_e, s_{-e}\right) = -\sum_{p\in P}\sum_{q\in P}\sum_{f\in E\setminus e}\left[t_{p_e} t_{q_f} I_{ef}\left(s_e, s_f\right)\right]$$

$$V\left(r, s\right) = -\sum_{e\in E}\sum_{f\in E}\sum_{p\in P}\sum_{q\in P} t_{p_e} t_{q_f} I_{ef}\left(s_e, s_f\right)/4$$

- Not claiming anything about redefining as joint routing / frequency selection
  - That changes the player set

Delay Without Link Interactions

With Interactions



Example Architecture Notes

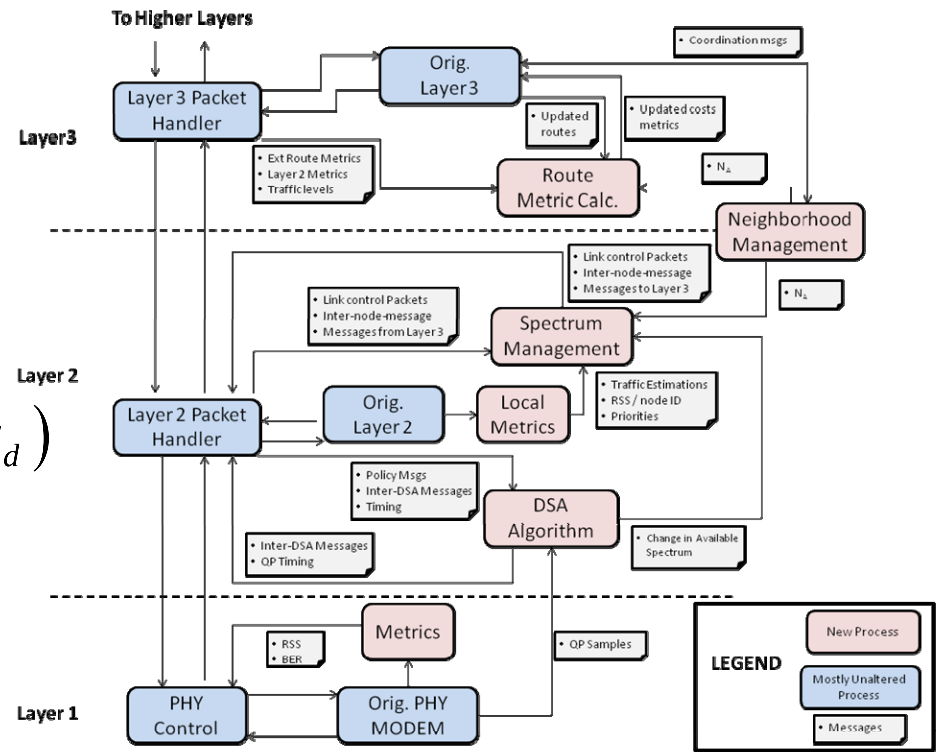
- Layer 3 Metric (edge cost)

$$c_e = \sum_{f \in N_e} t_e t_f I_{ef}\left(s_e, s_f\right)$$

- Spectrum Management

$$I_e\left(s\right) = \sum_{d \in E \setminus E_m} \sum_{n_1 \in e} \sum_{n_2 \in d} p_d g\left(n_1, n_2\right) \sigma\left(s_e, s_d\right)$$

- Power control
  - Target RSS (unilateral)
  - Need to know traffic levels on links where interacting
    - Different
  - Identification
    - Layer 2 (beacon)
    - Layer 3 (hops)



Complexity Estimations for Combination Techniques

- Techniques
  - Non-coupled
    - No interaction
  - Edgewise

$$c_{e,p}(s,r) = \sum_{f \in E \setminus E_m} t_e t_f I_{ef}(s_e, s_f)$$

  - Network wide
    - Product of potentials

- Comparative Evaluations
  - 1 is better than 3
- Computational complexity
- Timing sensitivity
- Messaging Overhead
- Performance
- Flexibility
  - General applicability

Approach	Computational Complexity	Timing sensitivity	Messaging Overhead	Performance	Flexibility
Orthogonal (O)	1	1	1	3	2
Edge-wise (E)	1a	2	1	2	3
Network-wide (N)	3	3	3	1	1

Addressing some practical issues

Impact of Noise

- Noise impacts the mapping from actions to outcomes, $f: A \to O$
- Same action tuple can lead to different outcomes
- Most noise encountered in wireless systems is theoretically unbounded.
- Implies that every outcome has a nonzero chance of being observed for a particular action tuple.
- Some outcomes are more likely to be observed than others (and some outcomes may have a very small chance of occurring)

DFS Example

- Consider a radio observing the spectral energy across the bands defined by the set *C* where each radio *k* is choosing its band of operation f_k.
- Noiseless observation of channel c_k

$$o_i(c_k) = \sum_{k \in N} g_{ki} p_k \theta(c_k, f_k)$$

- Noisy observation $\tilde{o}_i(c_k) = \sum_{k \in N} g_{ki} p_k \theta(c_k, f_k) + n_i(c_k, t)$
- If radio is attempting to minimize inband interference, then noise can lead a radio to believe that a band has lower or higher interference than it does

Trembling Hand ("Noise" in Games)

- Assumes players have a nonzero chance of making an error implementing their action.
  - Who has not accidentally handed over the wrong amount of cash at a restaurant?
  - Who has not accidentally written a "tpyo"?
- Related to errors in observation as erroneous observations cause errors in implementation (from an outside observer's perspective).

Noisy decision rules

- Noisy utility $\tilde{u}_i(a,t) = u_i(a) + n_i(a,t)$

Trembling Hand

Definition 4.20: *Friedman's Noisy Random Better Response* [Friedman_01]
Player i chooses an action $a_i \in A_i \backslash b_i$ where b_i is player i's current action according to a uniform random distribution. If $u_i(a_i,a_{-i}) > u_i(b_i,a_{-i})$, then a_i is implemented, however, if $u_i(a_i,a_{-i}) \le u_i(b_i,a_{-i})$, then player i still switches to a_i with nonzero probability ρ.

Observation Errors

Definition 4.21: *Noisy Best Response Dynamic* (*)
A decision rule $\tilde{d}_i : A \times T \to A_i$ is a noisy best response dynamic if each adaptation would maximize the radio's noisy utility if all other radios continued to implement the same waveforms, i.e., $\tilde{d}_i(a) \in \left\{ b_i \in A_i : \tilde{u}_i(b_i, a_{-i}, t) \ge \tilde{u}_i(a_i, a_{-i}, t) \forall a_i \in A_i \right\}$

Definition 4.22: *Noisy Better Response Dynamic* (*)
A decision rule $\tilde{d}_i : A \times T \to A_i$ is a noisy better response dynamic if each adaptation would improve the radio's utility if all other radios continued to implement the same waveforms, i.e., $\tilde{d}_i(a) \in \left\{ b_i \in A_i : \tilde{u}_i(b_i, a_{-i}, t) > \tilde{u}_i(a_i, a_{-i}, t) \right\}$.

Definition 4.23: *Noisy Random Better Response Dynamic* (*)
A decision rule $\tilde{d}_i : A \times T \to A_i$ is a random better response dynamic if for each $t_i \in T_i$, radio i chooses an action from A_i with nonzero probability and implements the action if it would improve $\tilde{u}_i$.

Implications of noise

- For random timing, [Friedman] shows game with noisy random better response is an ergodic Markov chain.
- Likewise other observation based noisy decision rules are ergodic Markov chains
  - Unbounded noise implies chance of adapting (or not adapting) to any action
  - If coupled with random, synchronous, or asynchronous timings, then CRNs with corrupted observation can be modeled as ergodic Makov chains.
  - Not so for round-robin (violates aperiodicity)
- Somewhat disappointing
  - No real steady-state (though unique limiting stationary distribution)

DFS Example with three access points

- 3 access nodes, 3 channels, attempting to operate in band with least spectral energy.
- Constant power
- Link gain matrix

g_{ik}	1	2	3
1	1	0.5	0.1
2	0.5	1	0.3
3	0.1	0.3	1

- Noiseless observations

(f_1,f_1,f_1)	(f_1,f_1,f_2)	(f_1,f_2,f_1)	(f_1,f_2,f_2)	(f_2,f_1,f_1)	(f_2,f_1,f_2)	(f_2,f_2,f_1)	(f_2,f_2,f_2)	(f_2,f_1,f_1)
(0.6,0.8,0.4)	(0.5,0.5,0.0)	(0.1,0.0,0.1)	(0.0,0.3,0.3)	(0.0,0.3,0.3)	(0.1,0.0,0.1)	(0.5,0.5,0.0)	(0.6,0.8,0.4)	(0.0,0.3,0.3)

- Random timing

Trembling Hand

- Transition Matrix, p=0.1

P	(f_1,f_1,f_1)	(f_1,f_1,f_2)	(f_1,f_2,f_1)	(f_1,f_2,f_2)	(f_2,f_1,f_1)	(f_2,f_1,f_2)	(f_2,f_2,f_1)	(f_2,f_2,f_2)
(f_1,f_1,f_1)	0	1/3	1/3	0	1/3	0	0	0
(f_1,f_1,f_2)	1/30	3/10	0	1/3	0	1/3	0	0
(f_1,f_2,f_1)	1/30	0	9/10	1/30	0	0	1/30	0
(f_1,f_2,f_2)	0	1/30	1/3	3/5	0	0	0	1/30
(f_2,f_1,f_1)	1/30	0	0	0	3/5	1/3	1/30	0
(f_2,f_1,f_2)	0	1/30	0	0	1/30	9/10	0	1/30
(f_2,f_2,f_1)	0	0	1/3	0	1/3	0	3/10	1/30
(f_2,f_2,f_2)	0	0	0	1/3	0	1/3	1/3	0

- Limiting distribution

(f_1,f_1,f_1)	(f_1,f_1,f_2)	(f_1,f_2,f_1)	(f_1,f_2,f_2)	(f_2,f_1,f_1)	(f_2,f_1,f_2)	(f_2,f_2,f_1)	(f_2,f_2,f_2)
0.0161	0.0293	0.3846	0.0699	0.0699	0.3846	0.0293	0.0161

Noisy Best Response

- Transition Matrix, $\mathcal{N}(0,1)$ Gaussian Noise

P	(f_1,f_1,f_1)	(f_1,f_1,f_2)	(f_1,f_2,f_1)	(f_1,f_2,f_2)	(f_2,f_1,f_1)	(f_2,f_1,f_2)	(f_2,f_2,f_1)	(f_2,f_2,f_2)
(f_1,f_1,f_1)	0.3367	0.2038	0.2381	0	0.2214	0	0	0
(f_1,f_1,f_2)	0.1295	0.4813	0	0.1854	0	0.2038	0	0
(f_1,f_2,f_1)	0.0953	0	0.6273	0.1479	0	0	0.1295	0
(f_1,f_2,f_2)	0	0.1479	0.1854	0.5548	0	0	0	0.1119
(f_2,f_1,f_1)	0.1119	0	0	0	0.5548	0.1854	0.1479	0
(f_2,f_1,f_2)	0	0.1295	0	0	0.1479	0.6273	0	0.0953
(f_2,f_2,f_1)	0	0	0.2038	0	0.1854	0	0.4813	0.1295
(f_2,f_2,f_2)	0	0	0	0.2214	0	0.2381	0.2038	0.3367

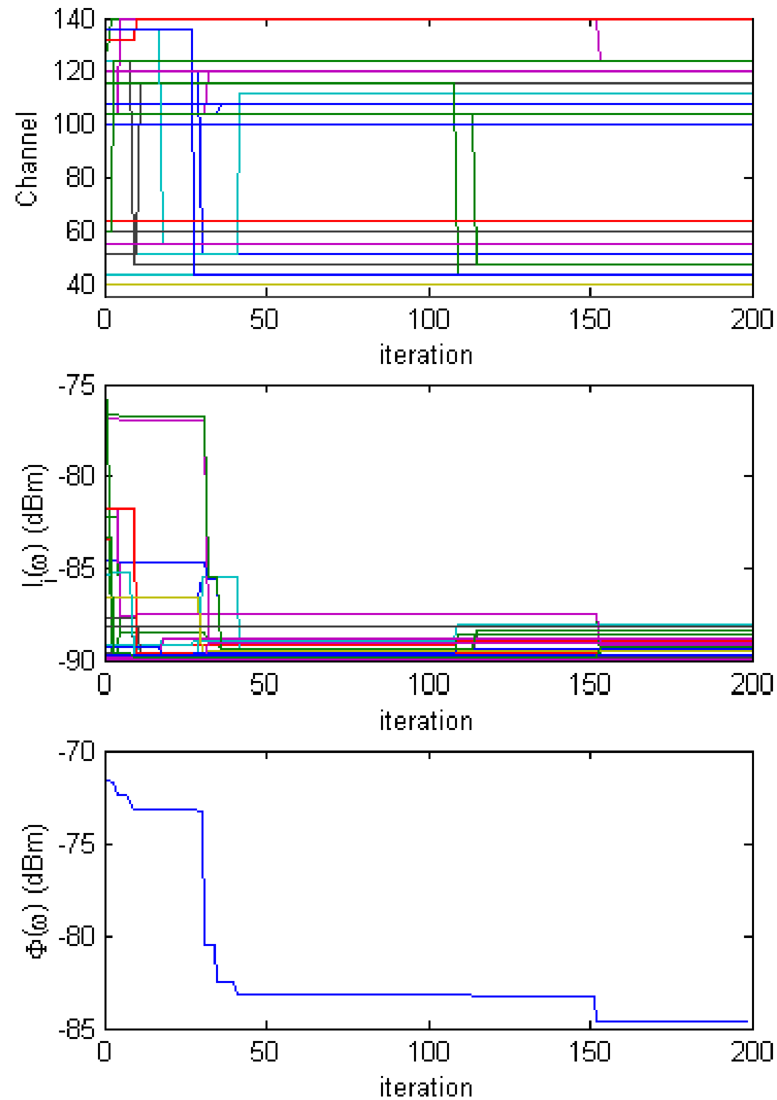
	(f_1,f_1,f_1)	(f_1,f_1,f_2)	(f_1,f_2,f_1)	(f_1,f_2,f_2)	(f_2,f_1,f_1)	(f_2,f_1,f_2)	(f_2,f_2,f_1)	(f_2,f_2,f_2)
σ=1.00	0.0709	0.1120	0.1765	0.1406	0.1406	0.1765	0.1120	0.0709
σ=0.50	0.0540	0.1040	0.1984	0.1436	0.1436	0.1984	0.1040	0.0540
σ=0.10	0.0129	0.0647	0.2857	0.1366	0.1366	0.2857	0.0647	0.0129
σ=0.05	0.0033	0.0397	0.3387	0.1183	0.1183	0.3387	0.0397	0.0033
σ=0.01	0	0.002	0.46	0.038	0.038	0.46	0.002	0

Comment on Noise and Observations

- Cardinality of goals makes a difference for cognitive radios
  - Probability of making an error is a function of the difference in utilities
  - With ordinal preferences, utility functions are just useful fictions
    - Might as well assume a trembling hand
- Unboundedness of noise implies that no state can be absorbing for most decision rules
- NE retains significant predictive power
  - While CRN is an ergodic Markov chain, NE (and the adjacent states) generally remain most likely states to visit
  - Stronger prediction with less noise
  - Also stronger when network has a Lyapunov function
  - Exception - elusive equilibria ([Hicks_04])

Somewhat Improved Process

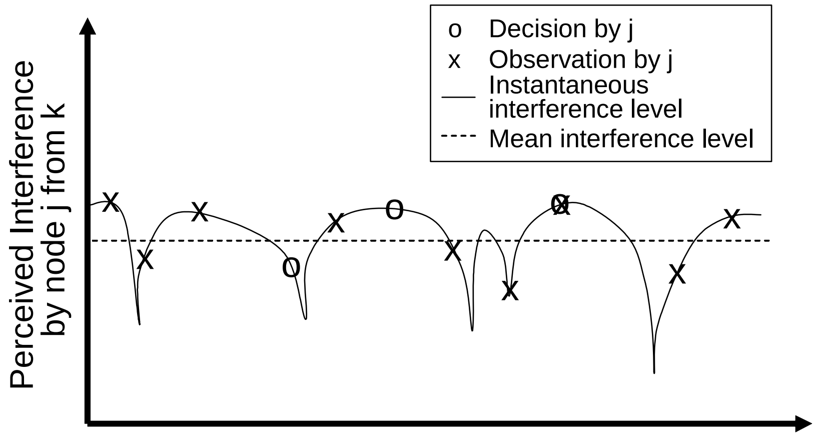
- Threshold adaptation (ε-better response)
  - Only adapt if adaptation expected to reduce interference by at least -85 dBm
- Stabilizes d_i:$O \rightarrow A$ not d_i:$A \rightarrow A$
  - Small variations in observations lead to same action



Stochastic Conditions

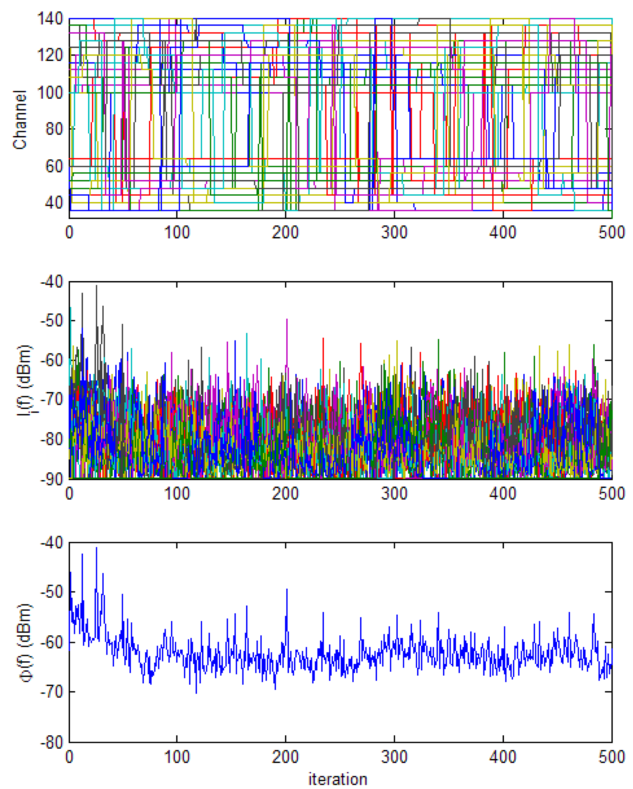
- Issue:
  - Unsynchronized observations and small scale fading leads to asymmetric interference

$$g_{jk}\left(t^{k}\right) \neq g_{jk}\left(t^{j}\right)$$



- Leads to unnecessary adaptations

30 Fixed P2P links with σ = 10 dB

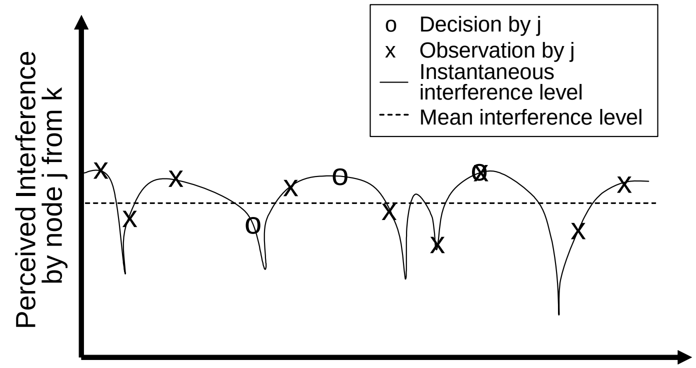


Solution

- Concept:
  - While sample values vary, they are samples of the same process
    - Implies same mean
    - Implies BSI in expected interference

$$E\left[g_{kj}p_k\rho\left(\omega_j,\omega_k\right)\right]=E\left[g_{jk}p_j\rho\left(\omega_k,\omega_j\right)\right]$$

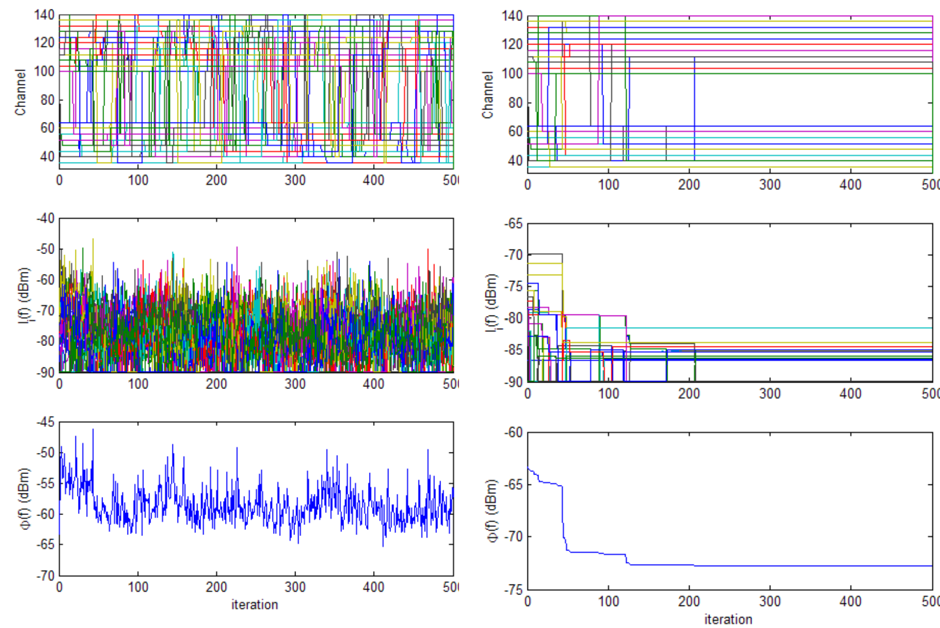
- Estimate mean for each observed device
  - Complexity grows with density



Without Mean Estimation

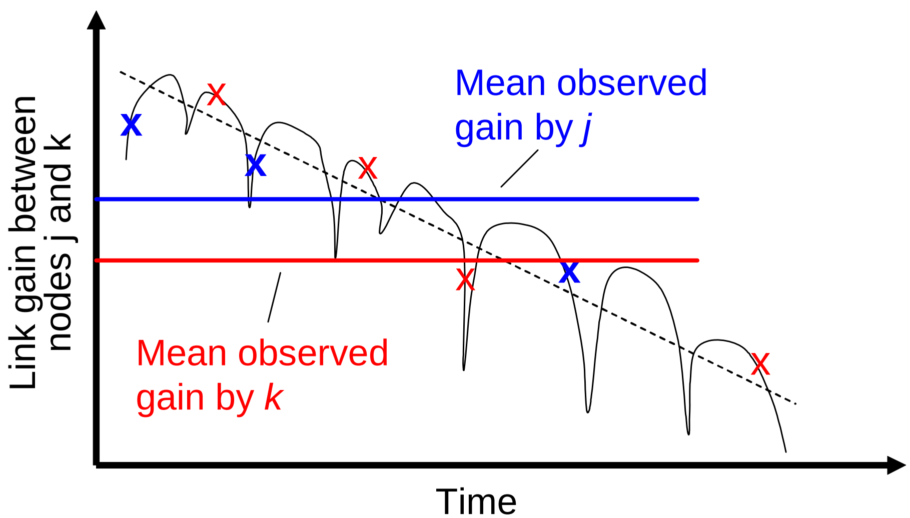
σ=10 dB

With Mean Estimation



Mobile Devices

- Issue:
  - Mobile devices imply non-stationary means
  - Mean estimate by one device != estimate by another device
  - Violates symmetry



- Solution concept:
  - Use Kalman filter to estimate moving average mean
  - Project forward to time of decision for estimated BSI equality
- Explicitly estimate large scale path loss model (and parameters) while "filtering" away small-scale path loss

Slight generalization on mismatched information and decision timings

- If information is stale, then bad decisions are made

	A	B
a	(0,0)	(1,1)
b	(1,1)	(0,0)

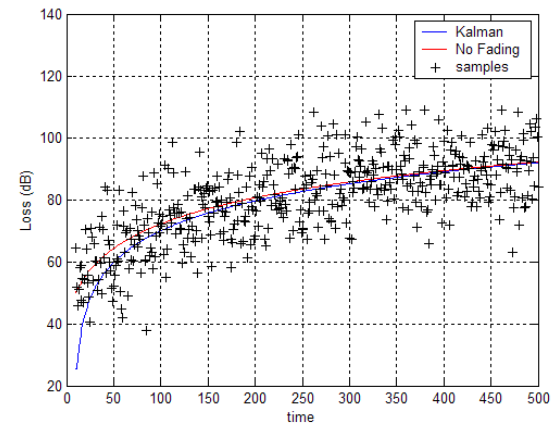
	A	B
a	(0,0)	(1,1)
b	(1,1)	(0,0)

- Want information update rate to be faster than decision rate
  - Bounds how fast you can respond
  - A motivation for why CRT doesn't do pure "joint" cross-layer algorithms

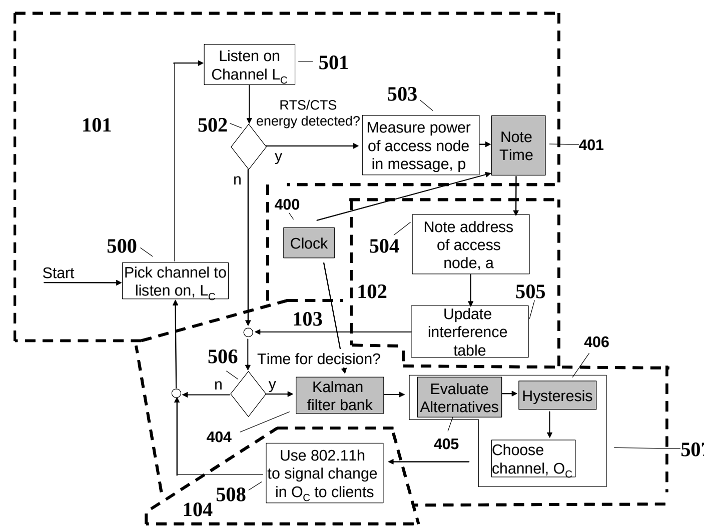
Mobility Solution

- Kalman Filter Design
  - –Inputs:
    - RSS associated with devices (implies ID recovery)
    - Observation times
  - –Hidden states:
    - Distance, velocity, change in velocity, path loss exponent, free-space loss / distance
  - –Outputs:
    - Estimated / projected mean large scale path loss
    - Hidden states
- Makes for a "one-size-fits-all" solution (v = 0) for fixed and mobile networks
- Complexity function of density

Example Estimation of Mobile Channel

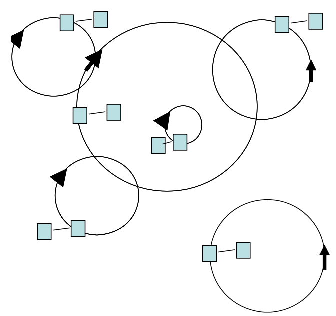


Example Integration

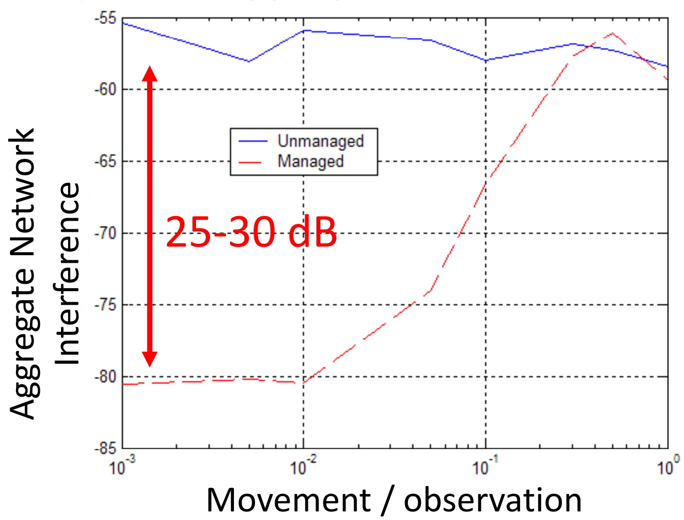


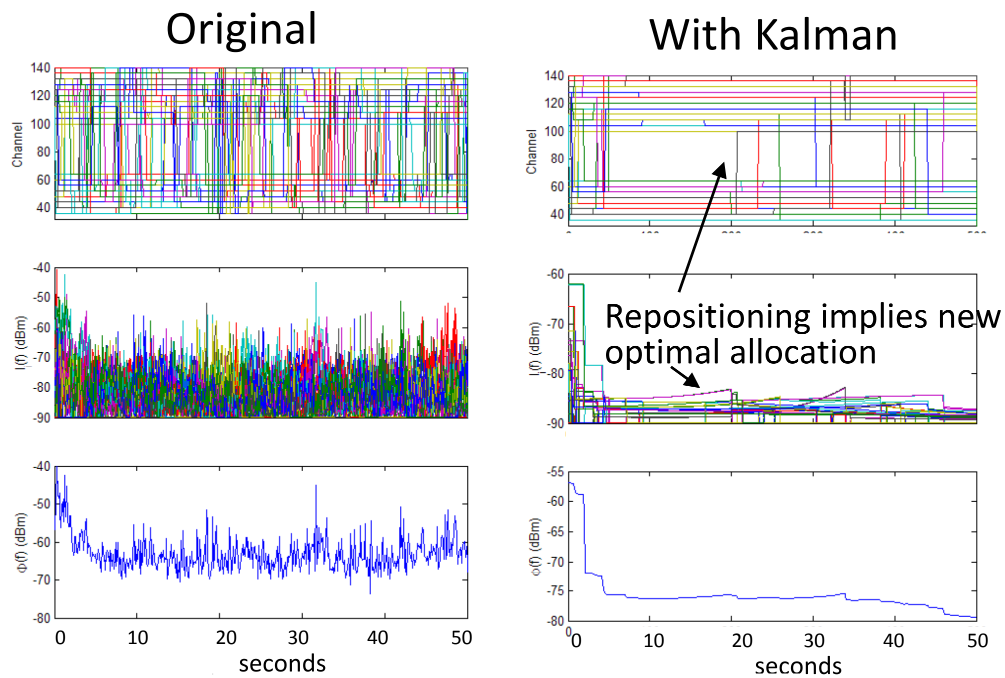
Mitigating instability induced by dynamic multipath environments

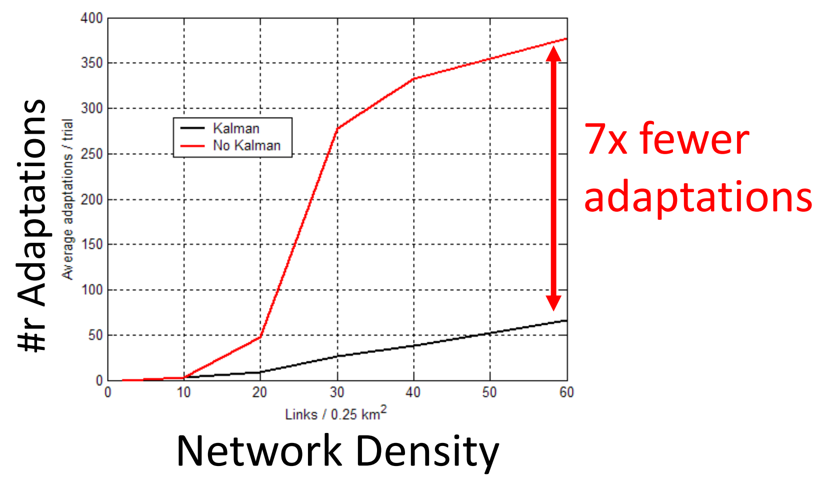
- Notional mobility model (random velocities, radii)



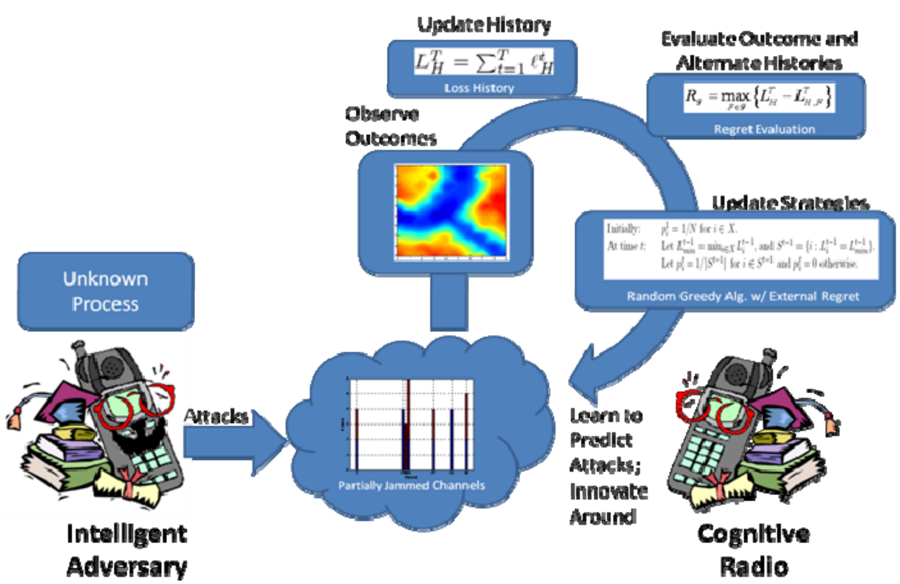
Steady-state aggregate interference





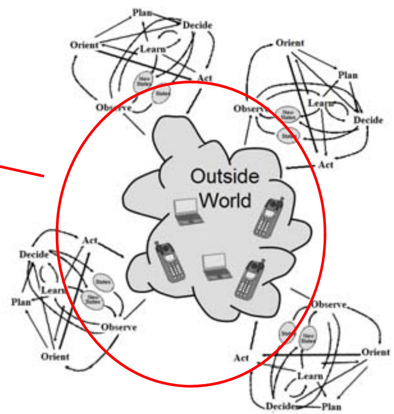


Adversarial Games



Hostile users can create problems from outside your network

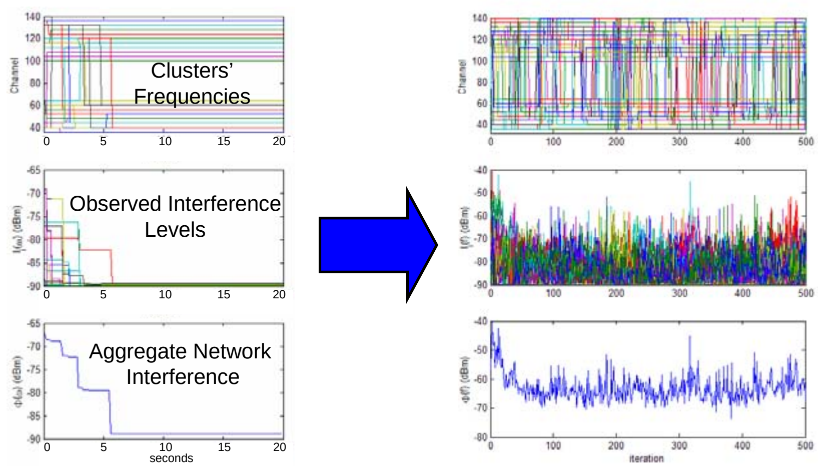
What if the environment is "unstable"?

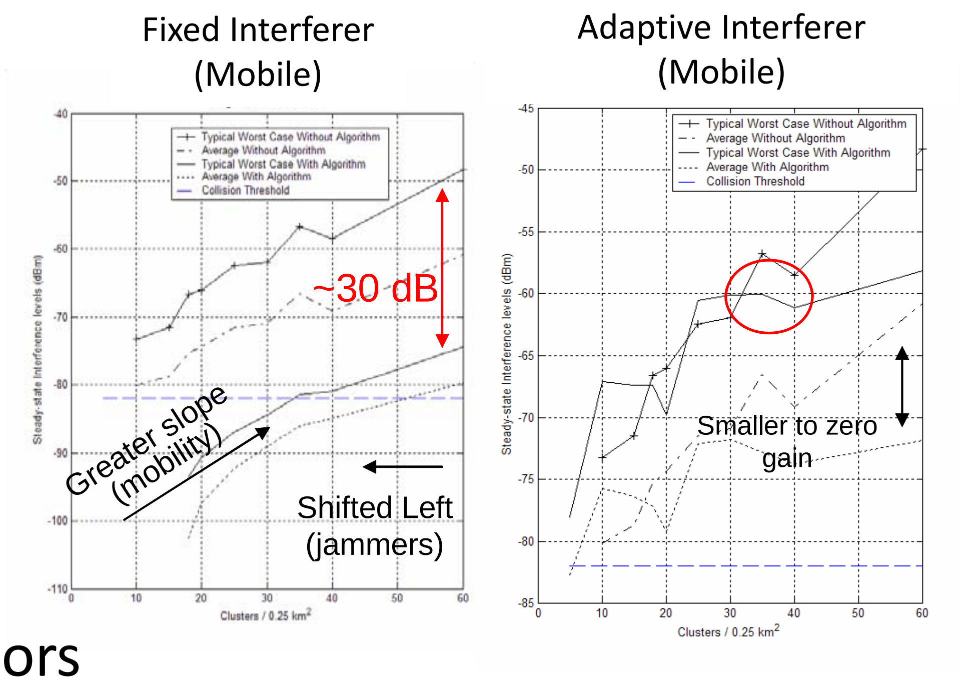


- Suppose another network is compromised in your area
- Their behavior influences your network's adaptations

- Performance Impact

- Stability impact



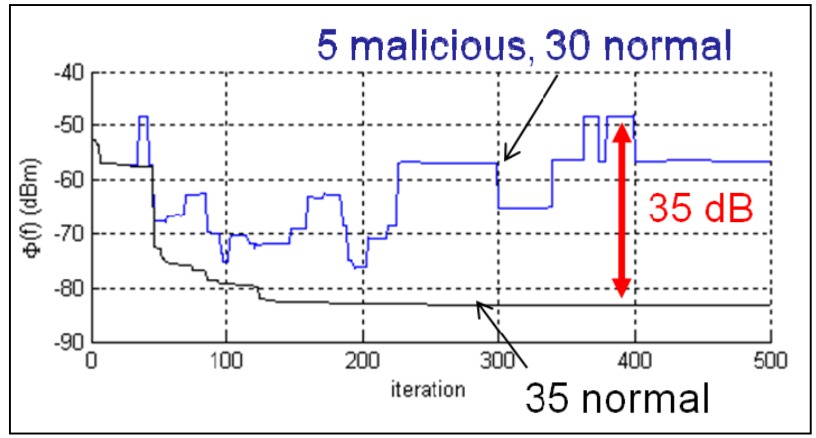


- Need to consider external actors
  - Detect unexpected behavior, adjust accordingly

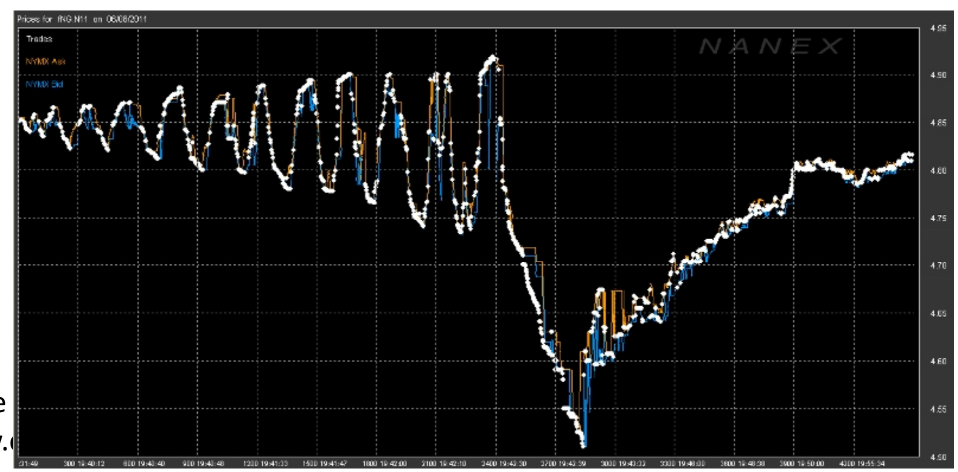
Known adaptation behaviors can be exploited by hostile users

- Observational attacks
  - Spectrum sensing data falsification [Chen_08a]
  - Replay sensing attacks [Bian_08]
  - Quiet period jamming [Bian_08]
- Orientation attacks
  - Primary User Emulation
  - Honeypot attacks [Newman_09]
  - Chaff point attacks [Newman_09]
- Behavioral attacks
  - Induce instability
  - Minimize performance
  - Adapt at inopportune times
- Learning exploits
  - And spoofing
    - And information corruption

Impact of mixing in performance minimizers



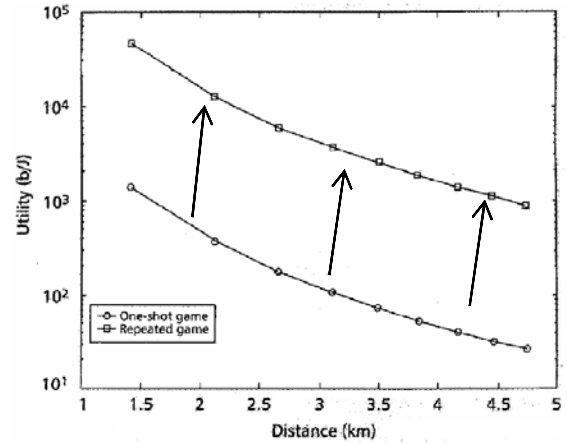
Destabilizing attack on natural gas exchange



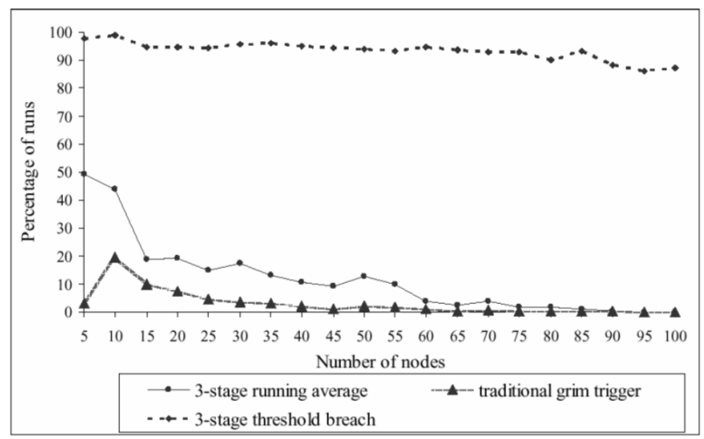
Cognitive
www.

Malicious != Selfish

- Popular "solution" to mischievous nodes (selfish nodes that damage network) is to "punish" nodes
  - Also implies a way to "brainwash" learning nodes
- Imperfect information can obfuscate punishment from mischievous behavior and produce catastrophic cascades
  - Brittleness
- Even with perfect information, malicious node may be masochistic



From Fig 6 in [MacKenzie_01]



From [Srivastava_06]

Zero-Sum Games

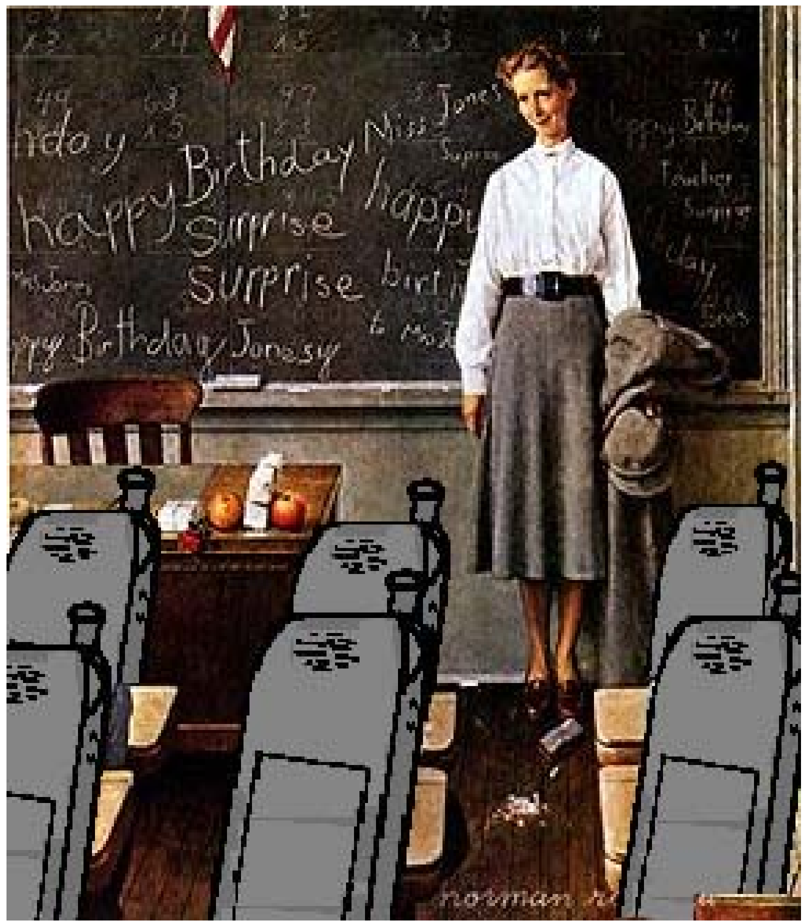
- To “win”, the other (set of) player(s) must “lose”
  - Generalize to strictly competitive games
- Strategies
  - Assign probability to each action
  - Any fixed strategy will be defeated
  - Mixed strategies needed
- Play
  - Max-min / min-max strategies optimal for perfect information
  - Uniform probability distribution appropriate with no information
  - Non-uniform with information or when side-benefit
- Improving on max/min equilibria => Get inside OODA loop
  - Adapt faster
  - Predict adaptations
  - Detect type of opponent

Learning (Predict adaptations, Detect type of opponent)

Jammer

Transmitter	0	1
0	(-1,1)	(1,-1)
1	(1,-1)	(-1,1)

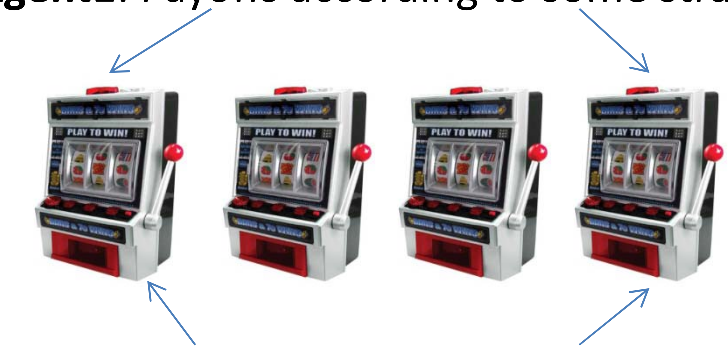
Learning in Games and Cognitive Radio



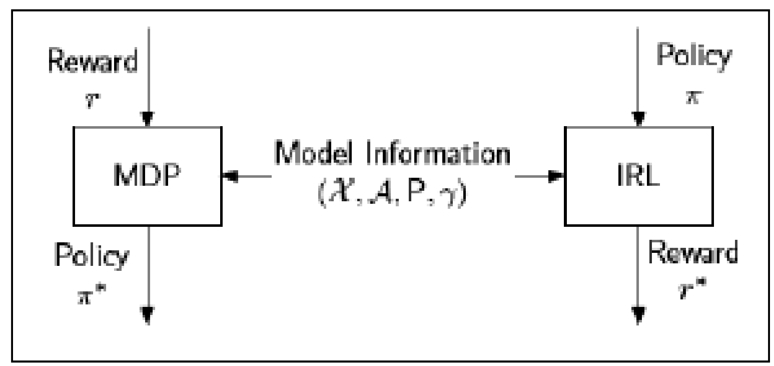
Approaches to Learning Key Game Model Elements

- Your own payoff and optimal strategies
  - Multi-armed bandit problems
  - Regret / No-regret learning
    - Really just a type of cost minimization that can be applied virtually anywhere
  - Reinforcement Learning
- Utility functions of other players
  - Inverse Reinforcement Learning
- Types of players and strategies
  - Bayesian Learning / Games

Agent1: Payoffs according to some strategy



Agent2: Learn strategy for optimal payout



Bayesian Games (Fudenberg)

- Augmented repeated normal form
- Players have private information (generally about their own type)
- Players share a common prior, e.g., distribution of types
  - Modeled as a choice by "Nature" (Imperfect information)
- Types are chosen by "nature" prior to play
  - Nature may have a "type"
- May or may not observe others' actions or payoffs
  - Some private signal
- Update beliefs on types according to Bayes's rule

$$\tilde{p}(\theta \mid m) = \frac{p(\theta)\sigma(m \mid \theta)}{p(m)} = \frac{p(\theta)\sigma(m \mid \theta)}{\sum_{\theta' \in \Theta} p(\theta')\sigma(m \mid \theta')}$$

Distribution of player types

$$\Theta = \{\Theta_1, \Theta_2, \ldots, \Theta_n\}$$

Beliefs on distribution of types

$$F : \Theta \to [0, 1]$$

Utility a function of types and actions

$$u_i : A \times \Theta \to \mathcal{R}$$

λ

	C	D
C	5, 5	0, 8
D	8, 0	1, 1

Type I

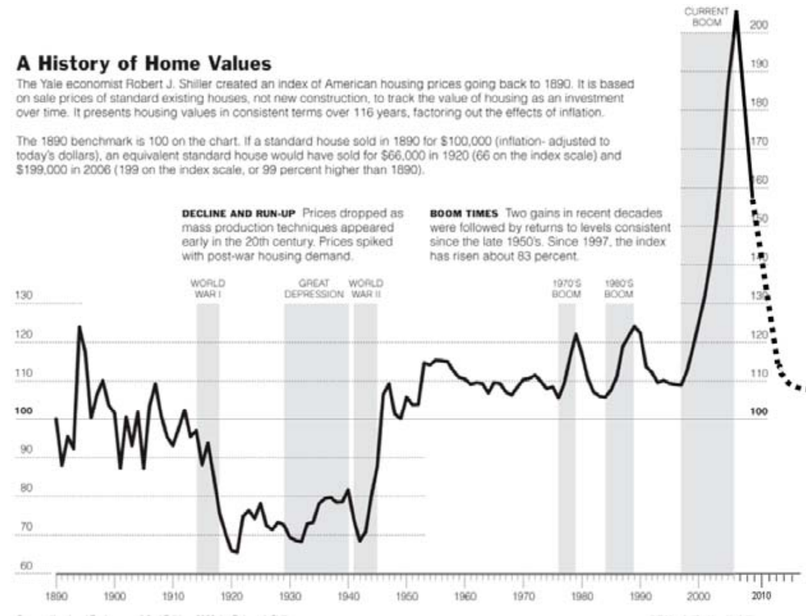
$1 - \lambda$

	C	D
C	5, 5	0, 2
D	8, 0	1, -5

Type II

Bayesian Games Notes

- Can have equilibria in beliefs that are not NE (and vice versa)
  - Actions are not observed or deduced
  - Different initial priors
  - NE agree on strategies, but distribution of actions also depend on types
- But there exist Bayesian games where equilibria in beliefs and NE coincide *only* if actions are *not* observed (Fudenberg)
- In general, not so easy to show convergence or stability nor reasonable equilbiria
- Impersonation can defeat type learning.



Instability in beliefs
(Also non-stationary Nature)

Fictitious Play Property (Monderer)

- Suppose the “type” is the stationary mixed strategy used by the player
- Actions are observed
- Beliefs are updated as the empirical distribution of past actions

$$b_k^i(t) = \frac{1}{t}\sum_{s=0}^{t-1} x^k(s),\ t \geq 1$$

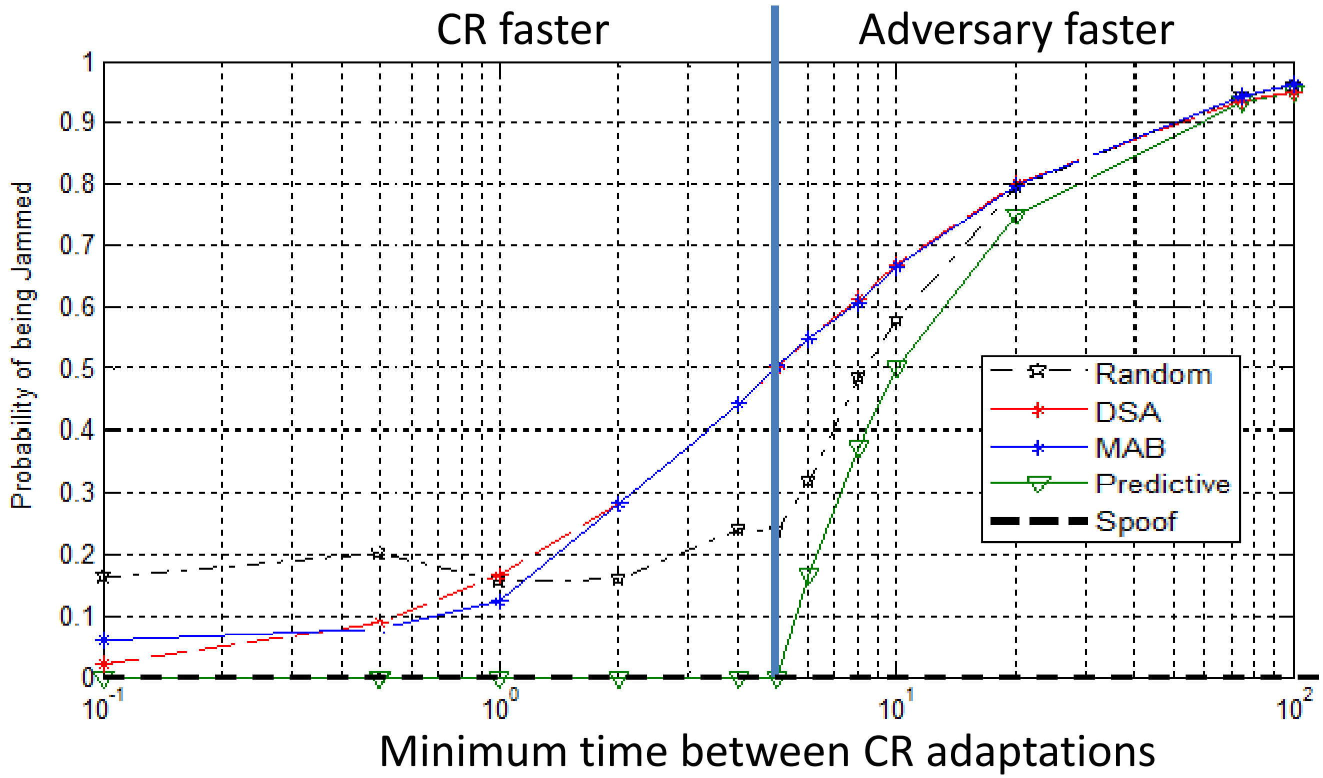
- If such a game always converges to an equilibrium independent of initial actions and beliefs, it is said to have FPP

- Games with FPP
  - 2 person zero-sum
  - 2 person 2x2 games
  - Dominance solvable
  - Linear Cournot
  - Coordination games
  - Best response equivalent in mixed strategies to a coordination game
  - Weighted Potential games

Population Games and Learning Strategies (Sandholm)

- Players get a chance to update their strategies at random intervals
  - Results shown for Poisson process
  - Assumes best response
- Different types (populations) of players have different available strategies
- Anonymity – payoffs depend only on population, strategy chosen, and number playing that strategy
- Payoffs perturbed by random shocks
- Potential games and supermodular games converge and are stable
  - Potential game result stronger than supermodular (must also be irreducible or larger set)
  - The same if unique NE
- Caveat:
  - With pertubations, system becomes ergodic
  - So really talking about neighborhood around equilibria that are visited with non-vanishingly small frequency
    - Lyapunov rest points

Example Result



- Random: open loop hopping
- MAB = multi-armed bandit solver
- DSA = react and choose least interfered
- Predictive = measure reaction rate and adapt ahead
- Spoof – Reverse PUE defense

Summary

What does game theory buy us in general?

- A natural “language” for modeling cognitive radio networks
- Permits analysis of non-algorithmic radios
  - Only know goals and that radios will adapt towards its goal
- Simplifies analysis of random procedural radios
- Permits simultaneous analysis of multiple decision rules – only need goal
- Provides condition to be assured of possibility of convergence for all autonomously myopic cognitive radios (weak FIP)
- Provides condition to be assured of convergence for all autonomously myopic cognitive radios (FIP, not synchronous timing)
- Rapid analysis
  - Verify goals and actions satisfy a single model, and steady-states, convergence, and stability
- An intuition as to what conditions will be needed to field successful cognitive radio decision rules.
- A natural understanding of distributed interactive behavior which simplifies the design of low complexity distributed algorithms

Why Design CR Networks with Potential Games?

- Predictable behavior
- Broad range of convergence and stability conditions
  - Myopic
    - Only has to be able to evaluate impact of own adaptations
  - Ideally decouple goal selection from decision rules
- Minimal communications
  - Looking for emergent behavior that coincides with design objective
- A lot of flexibility for individual radio customization
- Exact potential games tend to provide more mechanisms for combinations
- Suggest adapting / extending existing game models to new problems that approximate design objectives rather than working backwards from system design objective
  - Interference reducing networks for PHY algorithms (not power)
  - Edge congestion games for routing
- Gets messier as number of interacting algorithms increase
  - Orthogonal
  - Induce symmetry
  - Unidirectional dependencies

Adversarial Design is Less Satisfying

- Traditional maximin equilibria / solutions only apply when there are no timing advantages
  - And then you're only minimizing losses
- Key insights are ones that you already knew
  - Get inside the other guy's OODA loop
  - Else make it harder for them to observe your actions
    - Randomize
    - Spread / embed
    - Spoof

Questions you should ask before fielding your CRs

- Can you predict what will happen as it scales and interactions occur?
- How might your measures be turned against you?
  - Sensing, learning, policy enforcement
  - Even when following the “rules”
- How do you accommodate CR networks other than your own?
  - Can be attacked from outside without jamming
- What learning processes can your system accommodate and still behave appropriately.