PERFORMANCE OF DISTRIBUTED DYNAMIC FREQUENCY SELECTION SCHEMES FOR INTERFERENCE REDUCING NETWORKS

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ABSTRACT

One of the more commonly envisioned algorithms for cognitive radios is spectrum filling via dynamic frequency selection. Applying the cognitive radio design framework proposed in [1], we formalize a low complexity distributed ad-hoc dynamic frequency selection algorithm that converges to near-minimal interference frequency re-use patterns. We then examine the performance of this algorithm in the presences of practical considerations such as intra-network policy variations and timing issues and show that while this leads to situations that violate the framework of [1], the steady-state and convergence properties of the framework are still preserved.

INTRODUCTION[1]

When deployed in a network, the adaptations of cognitive radios yield an interactive decision problem which several authors have proposed modeling with game theory. By leveraging the potential game model, we proposed in [1] a framework – the *interference reducing network* (IRN) – for cognitive radio design that ensures the selfish adaptations of interacting cognitive radios converge to a low interference state. In brief, the framework requires each adaptation made by a cognitive radio to reduce the sum network interference. While it is easy to satisfy this condition with networks that employ centralized decision processes or elaborate observation sharing processes, this paper proposes a distributed and autonomous dynamic frequency selection algorithm (DFS) suitable for use in 802.11h that satisfies the IRN framework without cooperation between access nodes.

Many authors have attacked the problem of DFS, or more generally dynamic spectrum access (DSA), by requiring explicit coordination between access nodes. For instance, [2] considers a network of orthogonal channels where adaptive secondary users coordinate their adaptations via a common channel. [3] considers a system wherein optimal frequency/power allocations are achieved by employing punishment strategies. As part of a solution to network formation problem [4] utilizes a central controller to assign frequencies to each link in the network. [5] considers a DSA scheme wherein radios must share information over a common channel to compute the interference levels each radio would induce to other radios in order to evaluate its goal (U2 in [5]). While this has the virtue of being both an exact potential game and an IRN, it requires significant overhead to distribute the information needed to evaluate the goal and requires that decisions are made sequentially. For DSA systems where spreading codes adapted (viewed in the context of signal space, spreading code adaptation algorithms could be directly applied to DFS problems), [6] presents an algorithm where each radio's goal incorporates the interference measurements of all other radios in the system. [7-9] consider spreading code adaptations where each access node is isolated in frequency and spreading codes are chosen so as to minimize the interference of clients/mobiles are – a situation analogous in signal space to DFS applied to the clients in a single isolated cluster.

[5] also proposes another goal (or utility function) for DSA (U1) that is identical to the goal used in this paper (equation (1)). However, because [5] places no restrictions on the observation mechanism, [5] is unable to show that system forms an exact potential game which would permit the use of a simple distributed and autonomous algorithm. Instead [5] employs a no-regret learning algorithm wherein the radios autonomously try every possible frequency and then adapt to frequencies that yield the best weighted cumulative utility and show that the algorithm converges to a mixed strategy equilibrium – a less than optimal result as mixed strategies in frequency selection imply continuous probabilistic adaptation.

This paper proposes a low-complexity autonomous distributed DFS algorithm suitable for use in ad-hoc 802.11h networks. After briefly defining the concepts of

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interference reducing networks and exact potential games and defining the proposed algorithm, this paper shows via analysis and simulation that this algorithm results in a frequency allocation that is a minimizer of the sum of observed network interference even when different policies are applied to different channels, asynchronous decision timings are used, access nodes exhibit private frequency preferences, and spectral signals are imperfectly estimated. Additionally, the impact of combining transmit power control (TPC) with our DFS algorithm is explored.

INTERFERENCE REDUCING NETWORKS

Modifying the notation of [1] to be DFS specific, we can model a network of cognitive radios (or any goal-driven adaptive radios) by the tuple, $\langle N, F, \{u_i\}, \{d_i\}, T\rangle$ where N represents the set of n cognitive radios, F is the frequency space formed as $F = F_1 \times \cdots \times F_n$ where F_i specifies the frequencies available to cognitive radio $i \in N$, $\{u_i\}$, $u_i : F \rightarrow \mathbb{R}$, is the set of goals that inform the cognitive radios' decision processes, $d_i : F \rightarrow F_i$, implemented at the times that guides a radio's adaptations and the decision timings, T, at which the decisions are implemented. A cognitive radio network is said to be an *interference reducing network* (IRN) if all adaptations decrease the value of the sum of observed interference levels $\Phi(f) = \sum_{i \in N} I_i(f)$ where $I_i(f)$ is the interference observed by cognitive radio i when the frequency vector $f \in F$ is implemented by N.

For our DFS algorithm we'll model the goal of our radios as minimizing perceived interference as shown in (1)

$$u_i(f) = -I_i(f) = -\sum_{k \in N \setminus i} g_{ki} p_k s(f_i, f_k) \qquad (1)$$

where σ measures the fractional interference, i.e., $s(f_i, f_k) = \max\{B - |f_i - f_k|, 0\}/B$, f_i is the frequency of cognitive radio i's RTS/CTS signal, p_k is the transmission power of radio k's waveform, and g_{ki} is the link gain from the transmission source of radio k's signal to the point where radio i measures its interference. $\Phi(f)$ can then be expressed as in (2).

$$\Phi(f) = \sum_{i \in N} \sum_{k \in N \setminus i} g_{ki} p_k s(f_k, f_i) \qquad (2)$$

[1] states that an IRN can be realized in a distributed and autonomous fashion by selfish interference minimizing radios if adaptations are made by only one radio at a time if the condition of *bilateral symmetric interference* (BSI) holds which happens if $g_{ki} p_k s(f_i, f_k) = g_{ik} p_i s(f_k, f_i) \forall f_k \in F_k, \forall f_i \in F_i$. BSI implies that a network is an IRN for unilateral adaptations because BSI implies that $\langle N, F, \{u_i\}\rangle$ is an *exact potential game* [10]. An exact potential game is a normal form game for which there exists a function, called the *exact potential function*, $V : \Omega \rightarrow \mathbb{R}$, such that $u_i(\hat{f}_i, f_{-i}) - u_i(f_i, f_{-i}) = V(\hat{f}_i, f_{-i}) - V(f_i, f_{-i}) \forall i \in N, \forall f_i, \tilde{f}_i \in F_i$ where f_{-i} refers to the n-1 dimensional vector formed by excluding the contribution of i. By examining this definition, it is apparent that when selfish unilateral adaptations are made in an exact potential game, V constitutes a monotonically increasing sequence. When BSI holds, $\Phi(f) = -2V(f)$ [1], so a monotonically increasing V implies a monotonically decreasing $\Phi(f)$ making the network an IRN. This monotonicity property can then be used to prove the convergence of all selfish decision rules with unilateral timings [1].

A DFS IRN ALGORITHM

Consider a network of cognitive radios where each cognitive radio acts as an access node and observes the spectral energy of the RTS/CTS messages transmitted by the other access nodes in the network. [1] shows that if the network implements DFS under the following conditions, the network is an IRN:

C1: All messages are transmitted at the same power level.

C2: All adaptations made by $i \in N$ increase the value of (1) based on observations of the other cognitive radios' messages.

C3: All waveforms have the same bandwidth B.

C4: At any instance only a single radio adapts.

Note that C1 assures us that $p_k = p_i$, that C2 assures symmetric link gains between decision makers; C3 assures us that $s(f_i, f_k) = s(f_k, f_i)$. Thus BSI is satisfied. C4 then assures us of a monotonically decreasing $\Phi(f)$ when a radio's adaptations increase (1) which makes the network an IRN.

AN 802.11h APPLICATION

As [1] asserts, since the only requirement on the decision process of the cognitive radio is that adaptations increase (1) in order to decrease (2), great variation in the implementation of the decision process is permissible. In the following, we assume that each access node implements the following protocol:

1) Observe the spectral energy of the RTS/CTS messages of all observable access nodes.
2) At the time of its choosing, choose the channel on which the node observed the least amount of energy.

Consider a network of 802.11h access nodes (and presumably their client devices, but as the client devices are not involved in the decision process, they are irrelevant to the interactive decision problem). Suppose the access nodes are policy constrained to operate in the eleven channels available in the 5.47-5.725 GHz European band (channels 100-140) so that the assumption that “all RTS/CTS are transmitted at the same power level” holds for all channels (in this case, the band maximum of 1 W). Further, let us assume each radio has an equal probability of being the only radio allowed to adapt at each instance. As this is just a direct application of the general RTS/CTS DFS algorithm in [1] (where σ is now a binary function and discrete channels are used), we expect that the network will automatically sort itself into a low-interference frequency reuse pattern and that each adaptation will reduce the sum of observed interference in the network.

These expectations are confirmed in a simulation of thirty access nodes randomly distributed over 1 km^2 operating in an environment with a path loss exponent of 3 with random placements and random initial frequencies and noise powers of -90 dBm. The geographic distribution of devices and their final operating frequencies are shown in Figure 1 where a circle notes the position of an access node with its final channel id labeled just below and to the right of the circle. Figure 2 depicts the operational channels for each access node (top), perceived interference levels by the access nodes (middle), and the sum of perceived interference levels (bottom) for the simulated network. Note that $\Phi(f)$ (bottom) decreases with each adaptation thereby satisfying the definition of an interference reducing network even though there are instances of interference increasing for individual access nodes (middle). Thus as is the case for all IRNs, self-interested adaptations led to a socially desirable outcome (at least when socially desirable is defined as the sum of observed network interference levels).

POLICY VARIATIONS

If we permit the radios to choose permissible channels beyond channels 100-140, the assumption that all RTS-CTS messages are transmitted at the same power level fails as the lower and middle UNII bands (channels 36-64) limit transmission power levels to 200 mW [3]. This violates C1 ($p_k = p_i \ \forall i,k \in N$). However, for non-overlapping signals, $\sigma(f_i,f_k) = \sigma(f_k,f_i) = 0$, so BSI still holds and the network is still an IRN. Repeating the previous simulation and changing only the permissible channels and reflecting the transmission power policy variation we get the instantaneous statistics shown in Figure 3 where it is evident that the network continues to be an IRN.

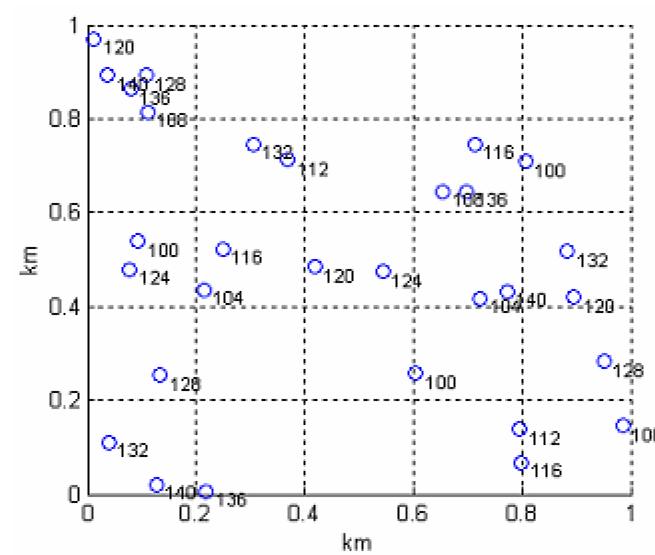

Figure 1: Steady-state Channels Selected for a Random Distribution of Access Nodes with Random Initial Channels in the 5.47-5.725 GHz Band.

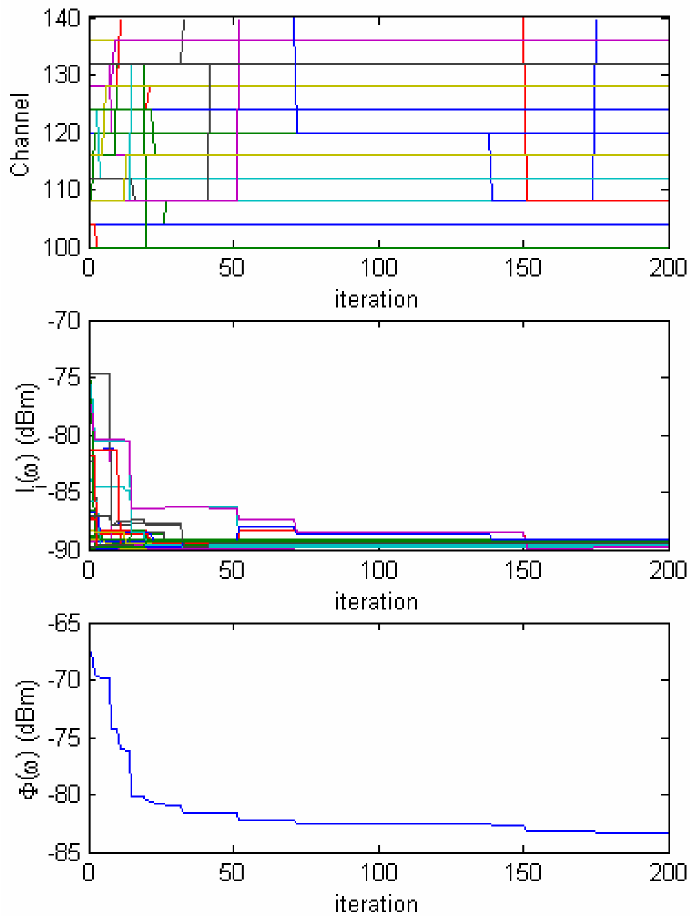

Figure 2: Instantaneous Statistics for Network in Figure 1

ASYNCHRONOUS TIMING

In the preceding, we assumed that one and only one access node adapted at any instance in time. However, because adaptations and observation processes do not occur in infintessimal periods of time it is likely that multiple

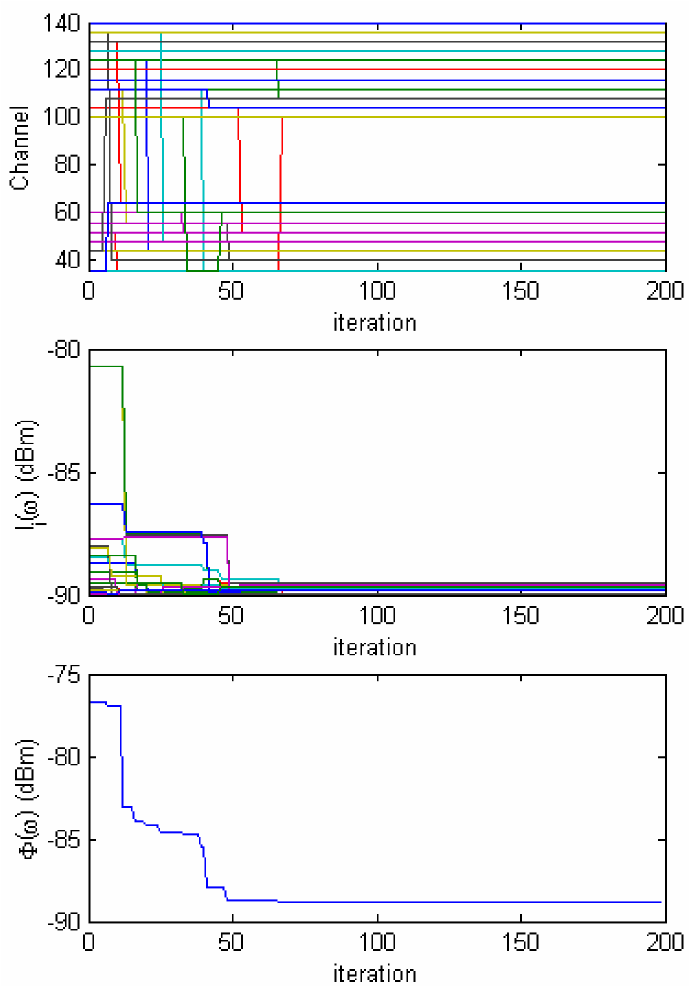

Figure 3: Instantaneous Statistics with Policy Variations

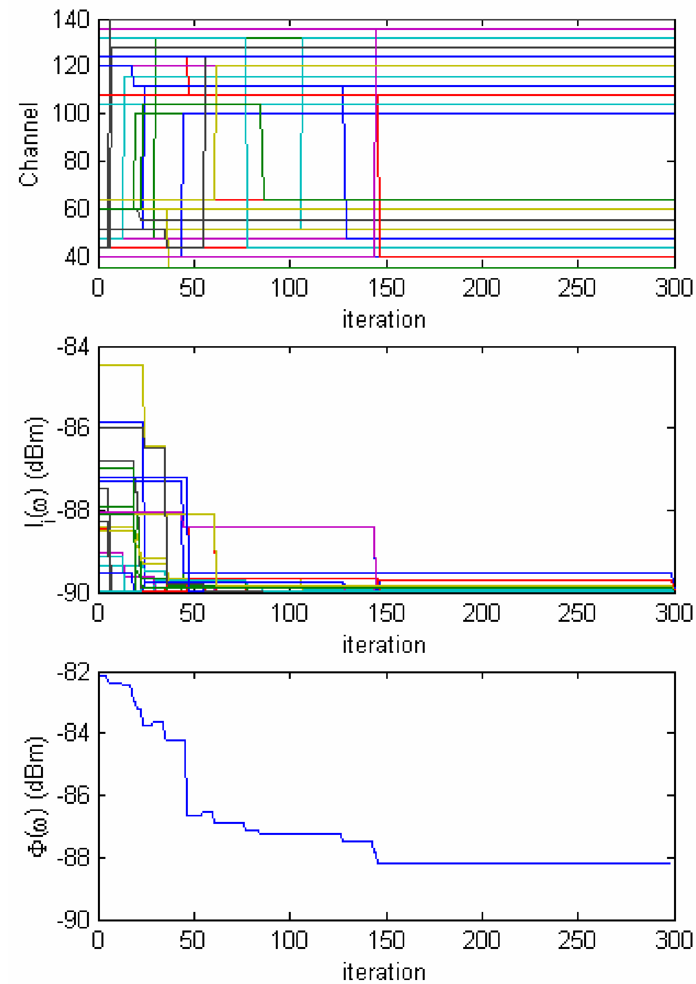

Figure 4: Impact of Asynchronous Decision Timings

access nodes will occasionally adapt simultaneously – a trend that becomes more likely as the number of access nodes in the network increase. So assuming C4 does not hold and continuing the policy violation of C1, we now assume each access has an opportunity to adapt at each iteration with non-zero probability.

Following the algorithm considered in this paper and the relaxed timing constraint two radios which are operating in the same channel and in close proximity to each other could simultaneously choose to adapt to another channel where a distant radio is operating. In this case, $\Phi(f)$ would increase even though each radio chose the channel which the radio had measured as having the least interference. Thus with C4 relaxed, the proposed algorithm cannot be guaranteed to yield the strict monotonicity required by the definition of an IRN.

Yet this network will still converge to a steady-state with that is a minimizer of $\Phi(f)$. This again is a result of $<N,F,\{u_i\}>$ forming an exact potential game. As it is an exact potential game, minimizers of $\Phi(f)$ are Nash equilibria and the game has the finite improvement path property which means that from any starting state, every sequence of self-interested unilateral adaptations must terminate in a minimizer of $\Phi(f)$ [2]. Due to these two properties, the network can be modeled as an absorbing Markov chain where minimizers of $\Phi(f)$ are the absorbing states of the chain. By virtue of being a minimizer, there can be no unilateral deviations that reduce interference; thus minimizers are absorbing states. By virtue of the finite improvement path property, there always exists a sequence of adaptations that terminate in a minimizer with non zero probability as long as the probability of a unilateral deviation is always nonzero. Thus even with C4 relaxed to asynchronous timings for adaptations, the network will still converge to a minimizer of $\Phi(f)$.

To verify this assertion, we modified the preceding simulation so that at each iteration each access node had an opportunity to adapt with probability 0.02. The instantaneous statistics for this simulation are shown in Figure 4. While $\Phi(f)$ still trends down, it is no longer doing so monotonically. Nonetheless, because this system forms an absorbing Markov chain, it eventually converges to a frequency vector that is a minimizer of $\Phi(f)$.

PRIVATE FREQUENCY PREFERENCES

Throughout this discussion we have assumed (C2) that each access node only intends to minimize the interference it perceives from other adaptive access nodes. However, because of the presence of interferers or because of local channel conditions, different access nodes may also exhibit different preferences for different frequencies. If we denote the frequency preferences of access node i as $S_i(f_i)$, these preferences might be incorporated as shown in (3).

$$\tilde{u}_i(f) = -\sum_{k \in N \setminus i} g_{kj} p_k s(f_i, f_k) - S_i(f_i) \tag{3}$$

Note that $S_i(f_i)$ indicates that this component for access node i is only a function of access node i's choice of frequency and makes the most sense express additively as in (3) when $S_i(f_i)$ models the influence of static interferers.

Under the assumption that $S_i(f_i)$ models static interferers in the environment (2) no longer reflects the sum network interference. Instead sum network interference with frequency preferences is given by (4).

$$\Phi^S(w) = \sum_{i \in N}\left(S_i(f_i) + \sum_{k \in N \setminus i} g_{ki} p_k s(f_k, f_i)\right) \tag{4}$$

This inclusion of additional interferers/jammers may also impact bilateral symmetric as the interferers may not be transmitting at the same power level as the cognitive radios or may be operating with differing bandwidths.

Regardless of the loss of bilateral symmetric interference due to variances in the static interferers, $\langle N, \Omega, \{u_i\} \rangle$ remains an exact potential game but with an exact potential function given by (5).

$$V^S(w) = -\sum_{i=1}^{n}\left(S_i(f_i) + \sum_{k=i+1}^{n} g_{ki} p_k s(f_k, f_i)\right) \tag{5}$$

Note that the differences between (4) and (5) imply that the network is not strictly an IRN. Consider the scenario where a unilateral adaptation is made from a channel that is originally only occupied by the adapting access node i and a static interferer to a channel that is occupied only by access node k such that (6) holds.

$$g_{kj} p_k s(f_i, f_k) < S_i(f_i) < 2 g_{kj} p_k s(f_i, f_k) \tag{6}$$

This adaptation would increase (3) – thereby satisfying the proposed algorithm – but (4) would also increase – violating the definition of an IRN. However, the exact potential in (5) will always increase, ensuring the algorithm's convergence. And when the only maximizers of (5) are those for which $S_i(f_i)=0\ \forall i \in N$, the algorithm will converge to a minimizer of (4) as for this condition $\Phi^S(f)= -2V(f)$. Even though it is trivial to construct two-access node, two channel, single interferer scenario with non-random geographic and channel distributions where (6) is satisfied, repeated trials of our randomly placed, random initial channel simulation have not yielded an adaptation that satisfies (6), which indicates the condition might be rare in practical settings. For example, modifying the policy variation simulation so it includes five static interferers operate in both channels 132 and 136, but distributed randomly geographically yield the simulation shown in Figure 5.

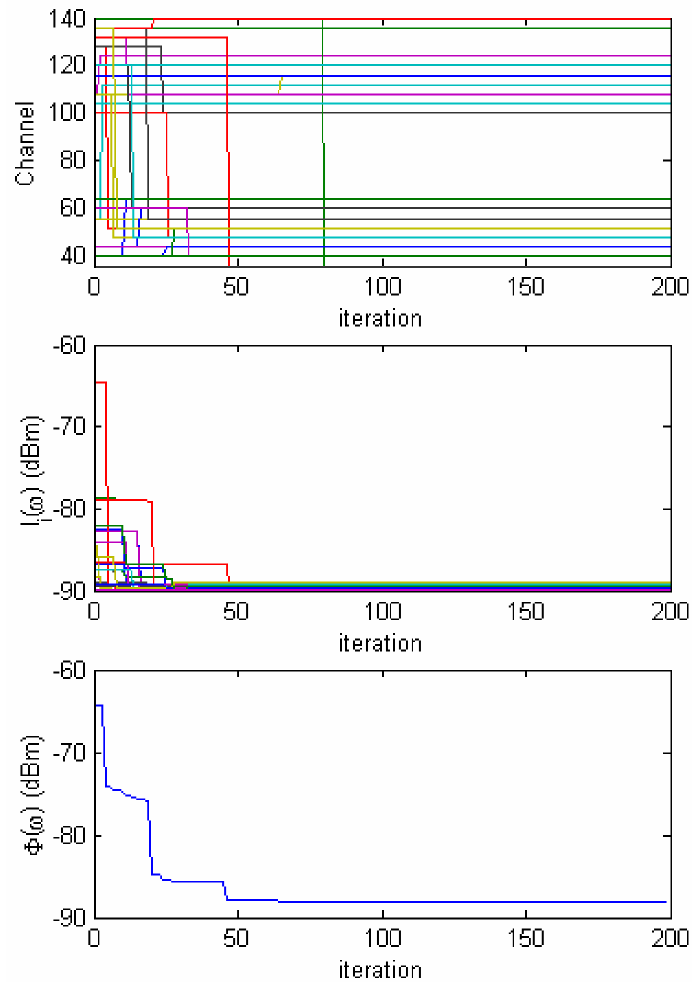

Figure 5: Algorithm with Private Frequency Preferences

EFFECT OF ESTIMATIONS

Throughout the preceding, we have implicitly assumed that the access nodes are perfectly measuring the signal strength of the RTS/CTS signals. However, in a practical setting, measurements of interference levels in differing channels would be corrupted by noise and thus only be estimations. In such a scenario, the access nodes' goals would again take the form as shown in (3) but with $S_i(f_i)$ a stochastic variable. As shown in the preceding section, a goal of the form of (3) implies that while $\langle N, F, \{u_i\} \rangle$ is still an exact potential game, the network will not necessarily remain an IRN for all possible realizations.

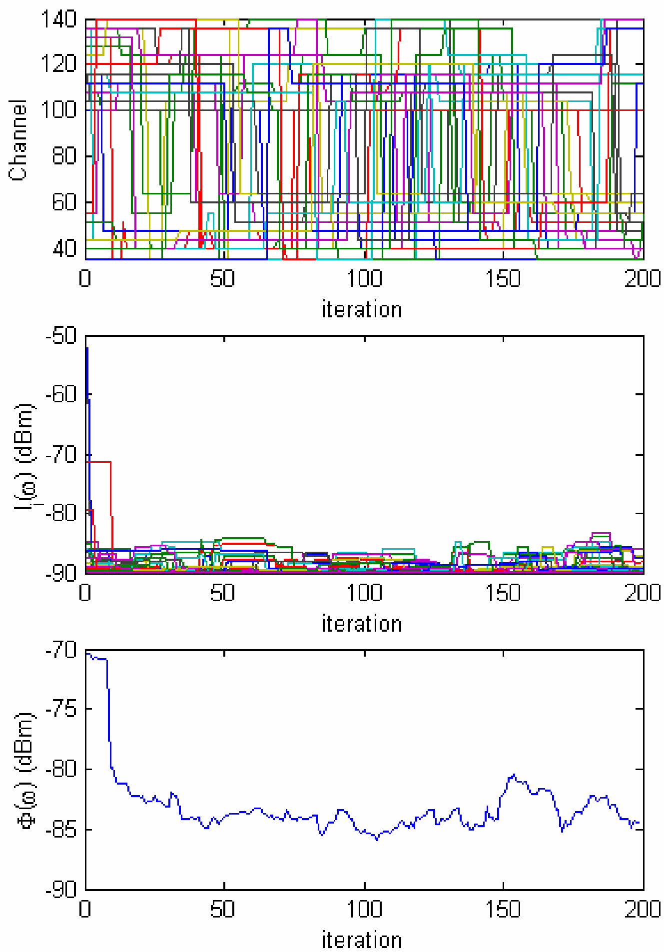

Figure 6 Algorithm With Stochastic Estimations

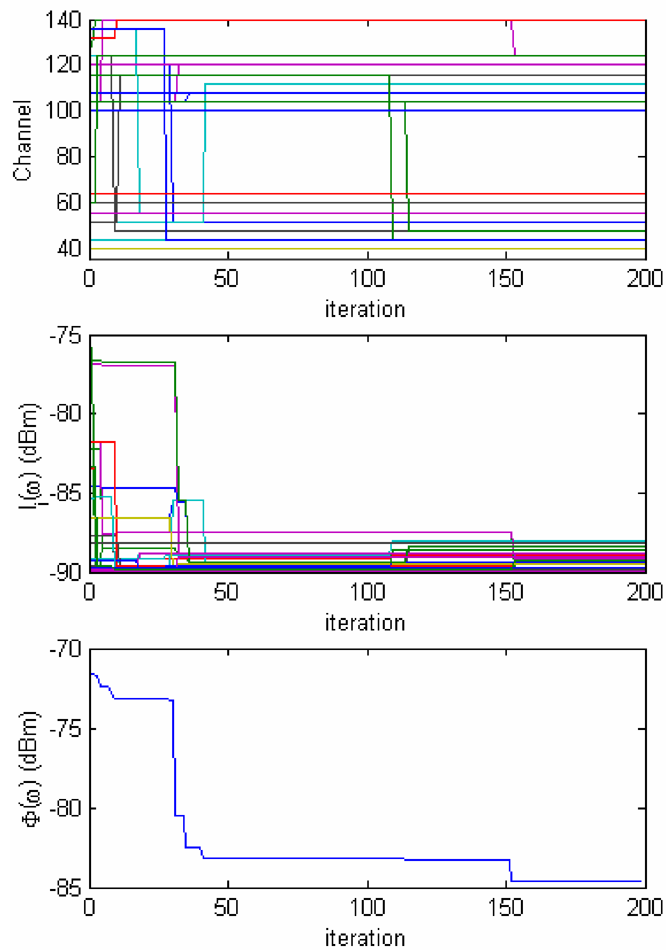

Figure 7 Algorithm with Stochastic Estimations and a small adaptation threshold (-85 dBm)

Further, for channels with very low interference levels, $S_i(f_i)$ may be a dominant term and its natural time variation may spawn unnecessary adaptations. For example consider a modification of the preceding simulation where the -90 dBm noise floor is implemented as a Gaussian stochastic variable whose results are shown in Figure 6. While the algorithm still yields an almost 15 dB reduction in interference levels from the initial random distribution, $\Phi(f)$ is no longer monotonic, overall performance is decreased and significant bandwidth would be wasted signaling all of these adaptations. However, by modifying the algorithm so the access nodes only adapt if the improvement in performance is predicted to be more than a small threshold (-85 dBm), the system behaves as shown in Figure 7 – generally like a convergent IRN, but with the caveat that there exists the small probability that an adaptation may increase sum interference.

TPC AND DFS

[3] states that TPC is intended to support variations in policy and adaptations based on "a range of information including path loss and link margin." As we showed in the Policy Variations section, as long as it is applied consistently across a channel policy variations do not impact the IRN features of the algorithm. However, if the RTS/CTS power levels are set at varying levels by the differing access nodes operating in the same channel, then it is likely that C1 will be violated in situations where $s(f_i, f_k) \neq 0$ which means the BSI condition will not be satisfied. For instance, consider a modification of the original policy variation simulation where the transmit power each access node applies to its RTS/CTS signals has been scaled by a factor randomly drawn from a clipped Gaussian distribution (no negative power levels) whose results are shown in Figure 8. Note that $\Phi(f)$ does not decrease monotonically in this simulation, though it does trend downwards fairly consistently and converges for all simulations to date. When TPC is applied to the RTS/CTS messages, it is observed that the system still converges to an interference minimizer. Currently, we do not have a firm analytic explanation for this phenomenon, though it is known that for relatively small variations in transmit power levels, $\Phi(f)$ will be an ordinal potential function (see [10] for an explanation of this term) for $\langle N, F, \{u_i\}\rangle$ so for many realizations of TPC applied to RTS/CTS

signals the network will still behave as an IRN. However, without a firm analytical basis for stating why desirable behavior results and as such are unable to rule out unforeseen pitfalls from the interactions, we recommend that application of the proposed algorithm be limited to scenarios where path loss based TPC is applied only to the DATA and ACK messages. While this assumption would still enable improved battery life and would be consistent with the RTS/CTS messages original intent for clearing out hidden nodes, it would limit the gains seen from frequency reuse. However, all localized TPC schemes face a functionality tradeoff of clearing out hidden nodes versus maximizing frequency reuse. By reducing transmit power on the RTS/CTS message, a higher cluster density can be achieved, but this comes at a cost of increasing the probability that a hidden node will miss the RTS/CTS signal and subsequently interfere with the data transfer, particularly where TPC is guided by local decisions instead of policy.

CONCLUSIONS

Leveraging the framework of interference reducing networks, this paper proposed a low complexity autonomous distributed ad-hoc DFS algorithm whose adaptations converge to a minimizer of the sum of observed interference levels by minimizing their own perceived interference measured from the RTS/CTS signals of other access nodes. We showed that this non-cooperative non-collaborative algorithm is robust to policy variations, timing variations, the presence of interferers, and noisy estimations of signal strengths when a simple adaptation threshold is applied to the algorithm. Though empirically convergent, when TPC is applied to the RTS/CTS signals, the algorithm fails to satisfy the IRN framework. However, the assumption of TPC applied to RTS/CTS signals may not be realistic as it necessarily increases susceptibility to hidden nodes. While all simulations implemented a best-response dynamic, any self-interested decision rule – including an ontological reasoning engine – will converge by virtue of being an exact potential game and an IRN.

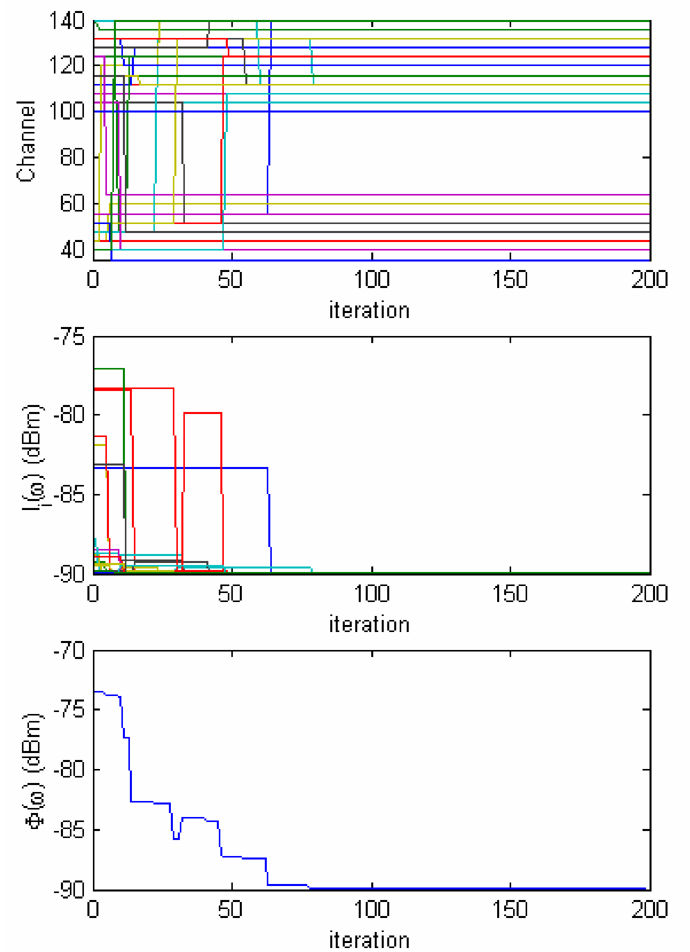

Figure 8 Algorithm with TPC Applied to RTS/CTS

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